Tutorial: Tensor Approximation in Visualization and Graphics

Scientific Visualization Applications

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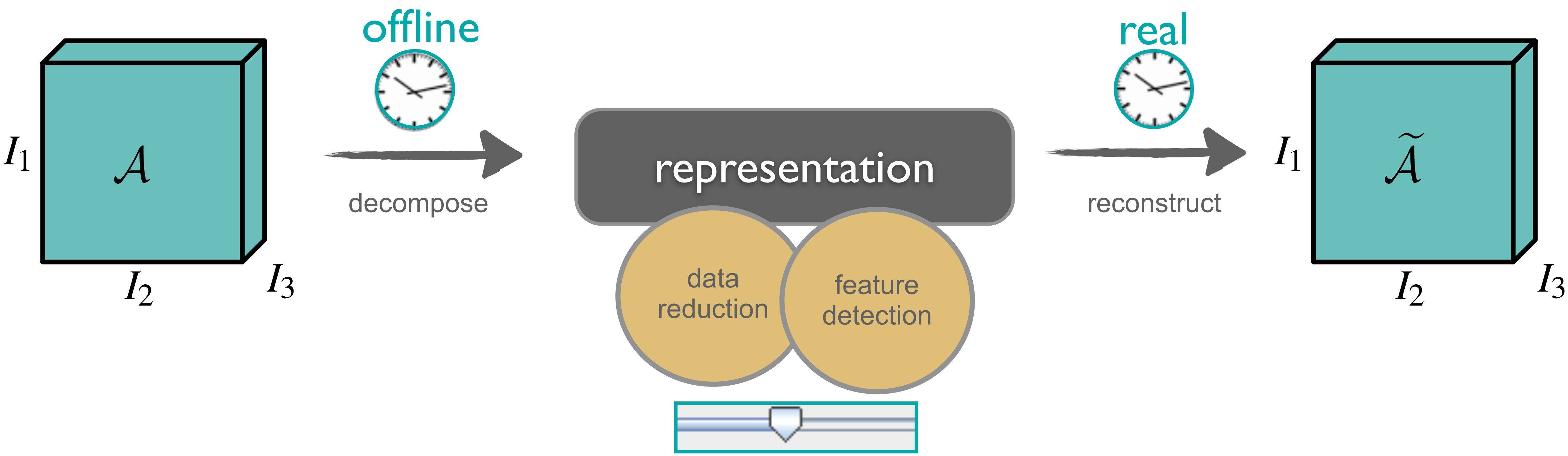
Computer Graphics

Outline

- Part 1: Compact data representations compared
  - wavelets
  - tensor approximation (Tucker model)
  - compression and multiscale features
- Part 2: Multiresolution TA Hierarchies

Part 1: Compact Data Representations Compared

Compact Data Representations



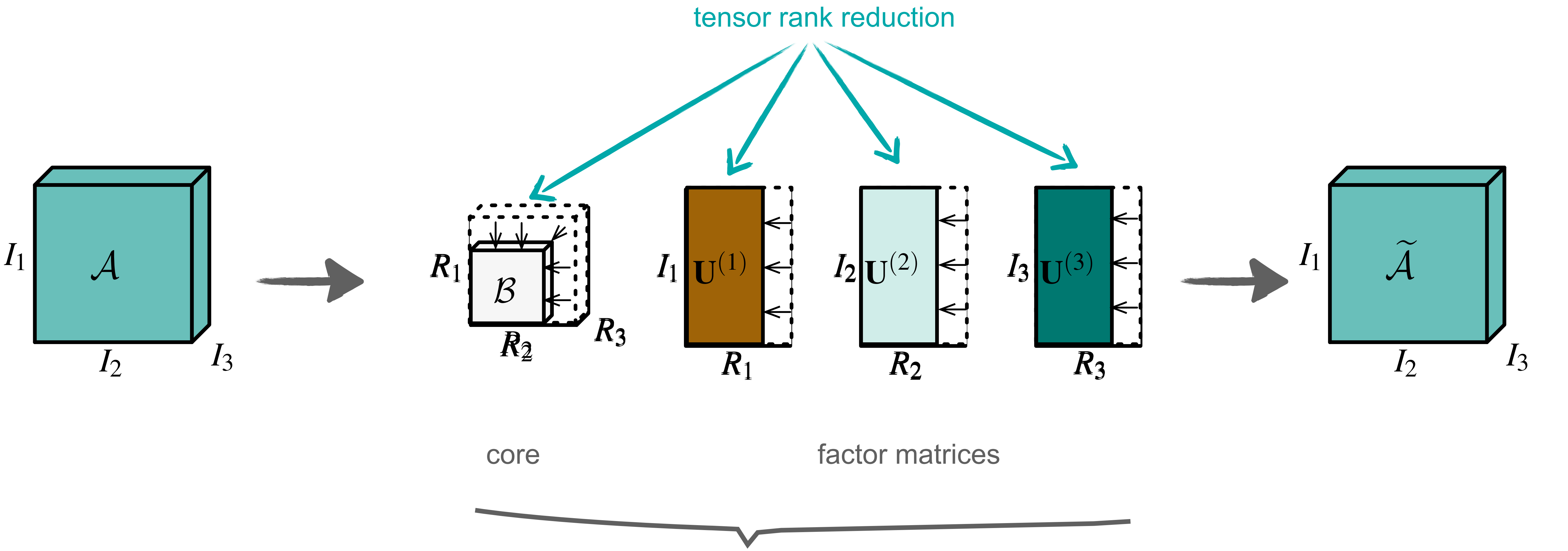
Compact Data Representation Models

- Discrete cosine transform
  [Yeo & Liu, 1995]
- Fourier transform
  [Chiueh et al, 1997]
- Wavelet transform
  [Rodler, 1999; Guthe et al, 2002]
- Vector quantization
  [Schneider & Westerman, 2003; Fout & Ma, 2007; Parys & Knittel, 2009]
- Tensor approximation
  [Tsai & Shih, 2006+2012; Wang et al., 2005; Wu et al, 2008; Suter et al., 2010+2011+2013]
- For details go to EG13 STAR on "A Survey of Compressed GPU Direct Volume Rendering"
  (Thursday, 11:00-12:40 in Room C.1)

Feature Extraction

- Typically done with multiresolution analysis
  - significant components at low frequencies
  - less important components at high frequencies
- Features at multiple spatial scales
- Multiscale feature extraction
  - achieved through tensor rank truncation

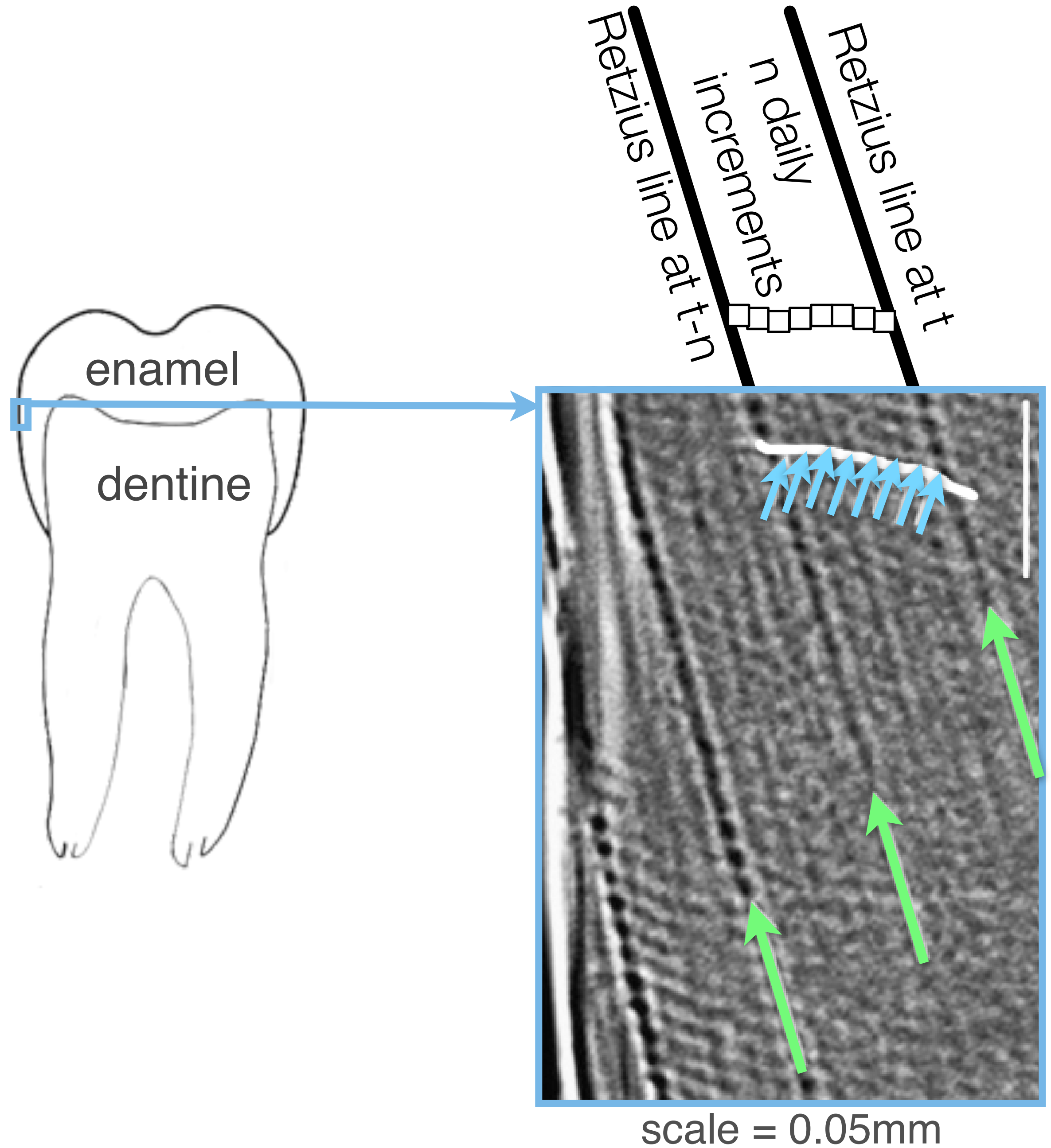
Tensor Rank Truncation (Tucker Model)



Tucker tensor decomposition

$$\widetilde{\mathcal{A}} = \mathcal{B} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \times_3 \mathbf{U}^{(3)}$$

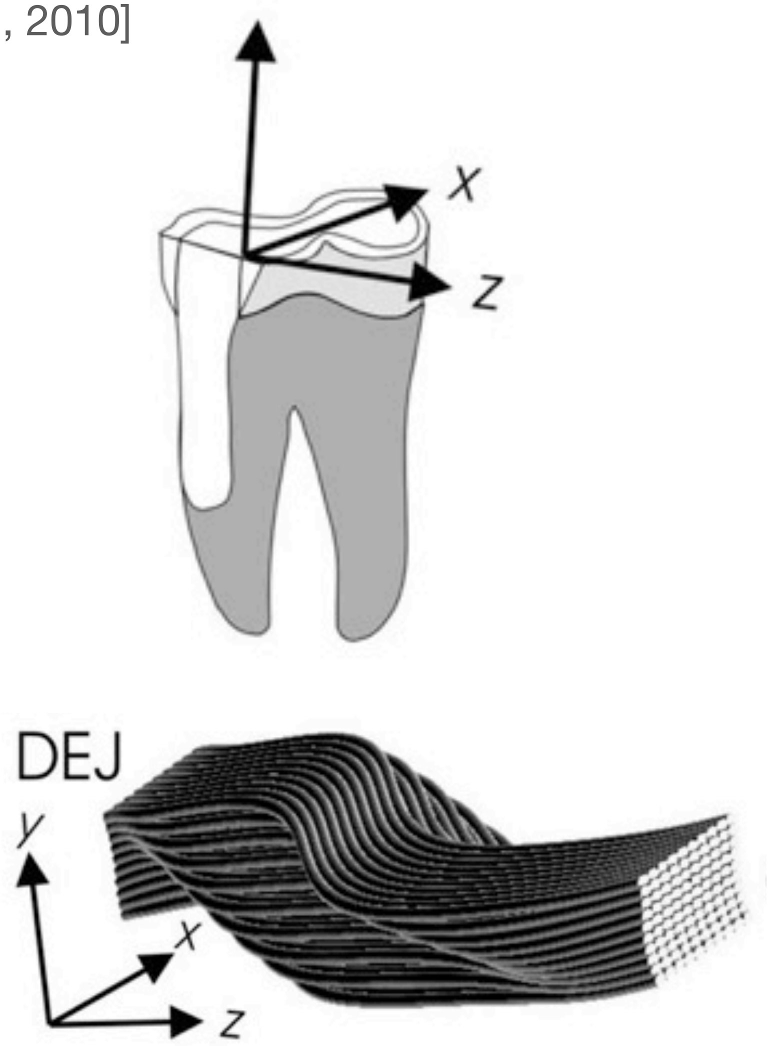
Application: Multiscale Dental Growth Pattern



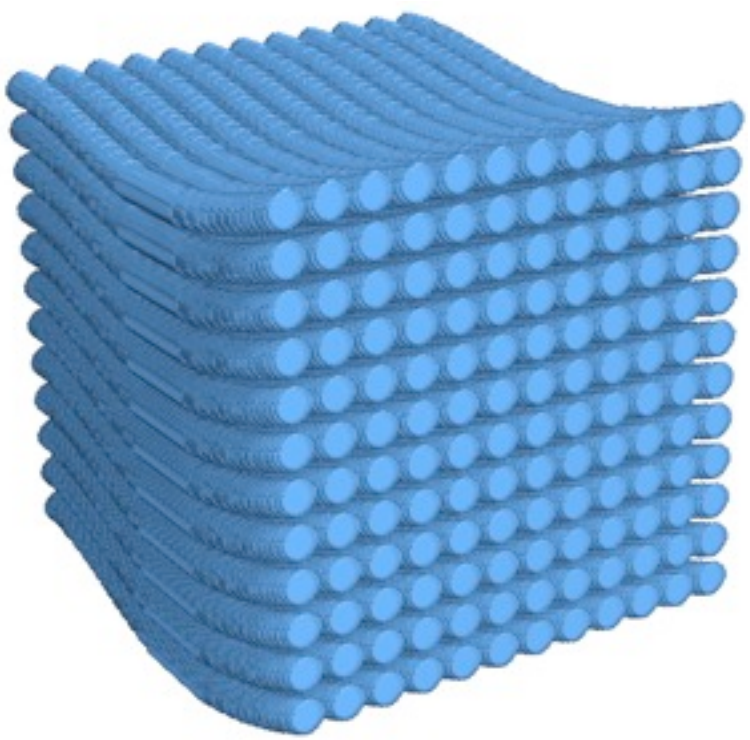
Suter, Zollikofer and Pajarola. Application of tensor approximation to multiscale volume feature representations. In *Proceedings Vision, Modeling and Visualization*, pages 203–210, 2010.

Synthetic 3D Dental Growth Structures

[Suter et al., 2010]

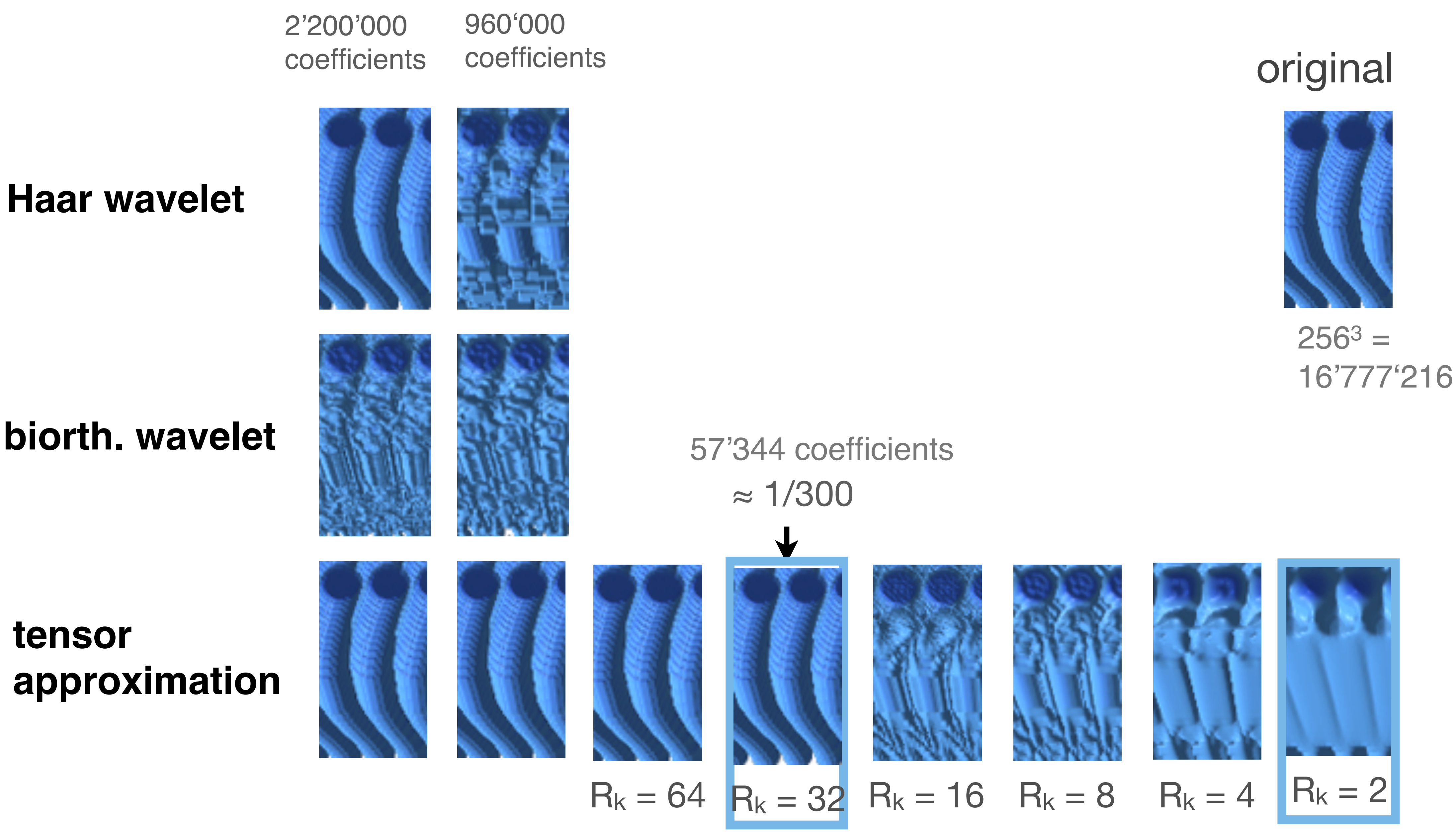


Source: Macho et al., 2003



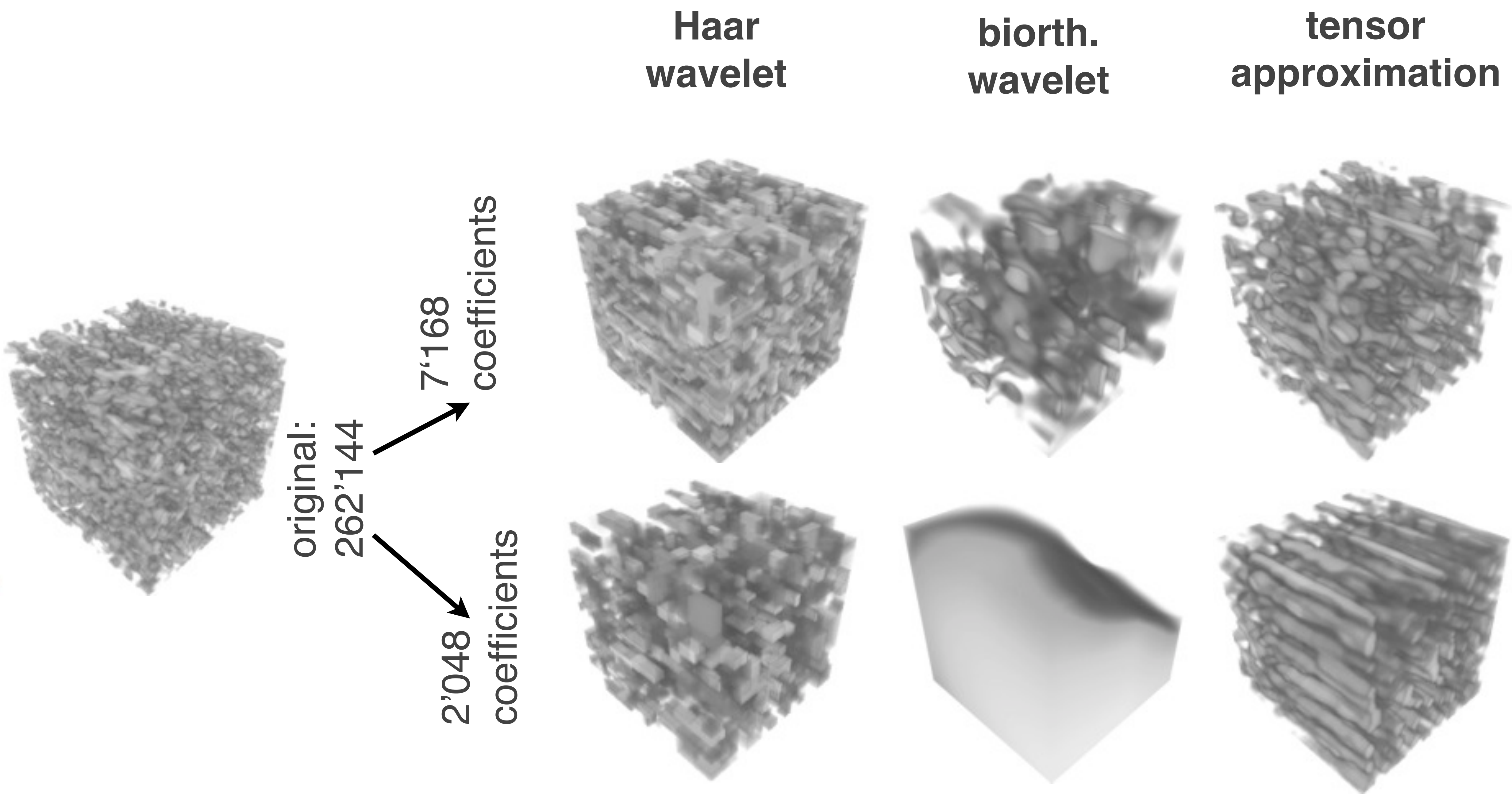
Non-axis-aligned Synthetic Features

[Suter et al., 2010]



Real Dental Growth Structures

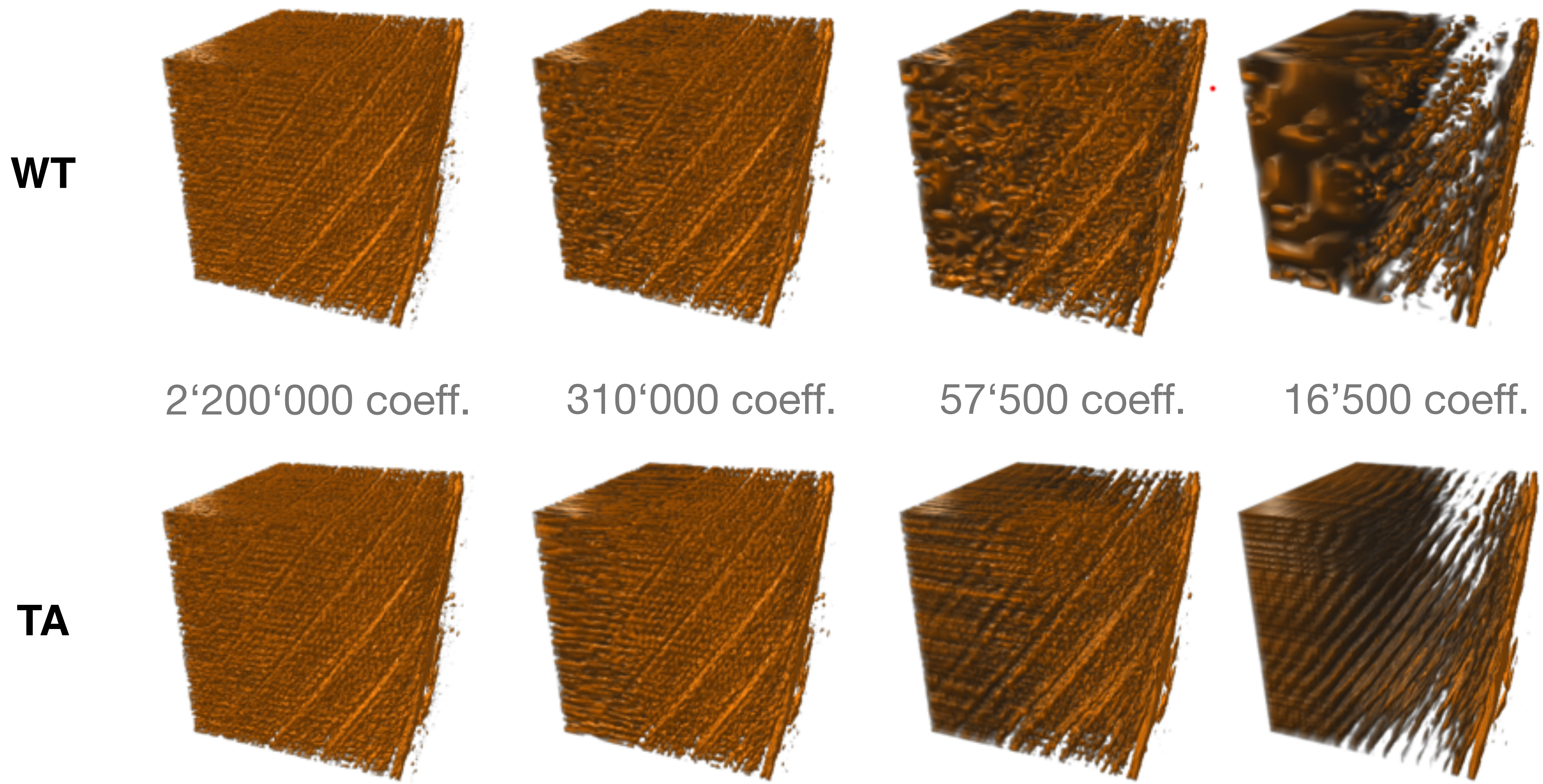
[Suter et al., 2010]



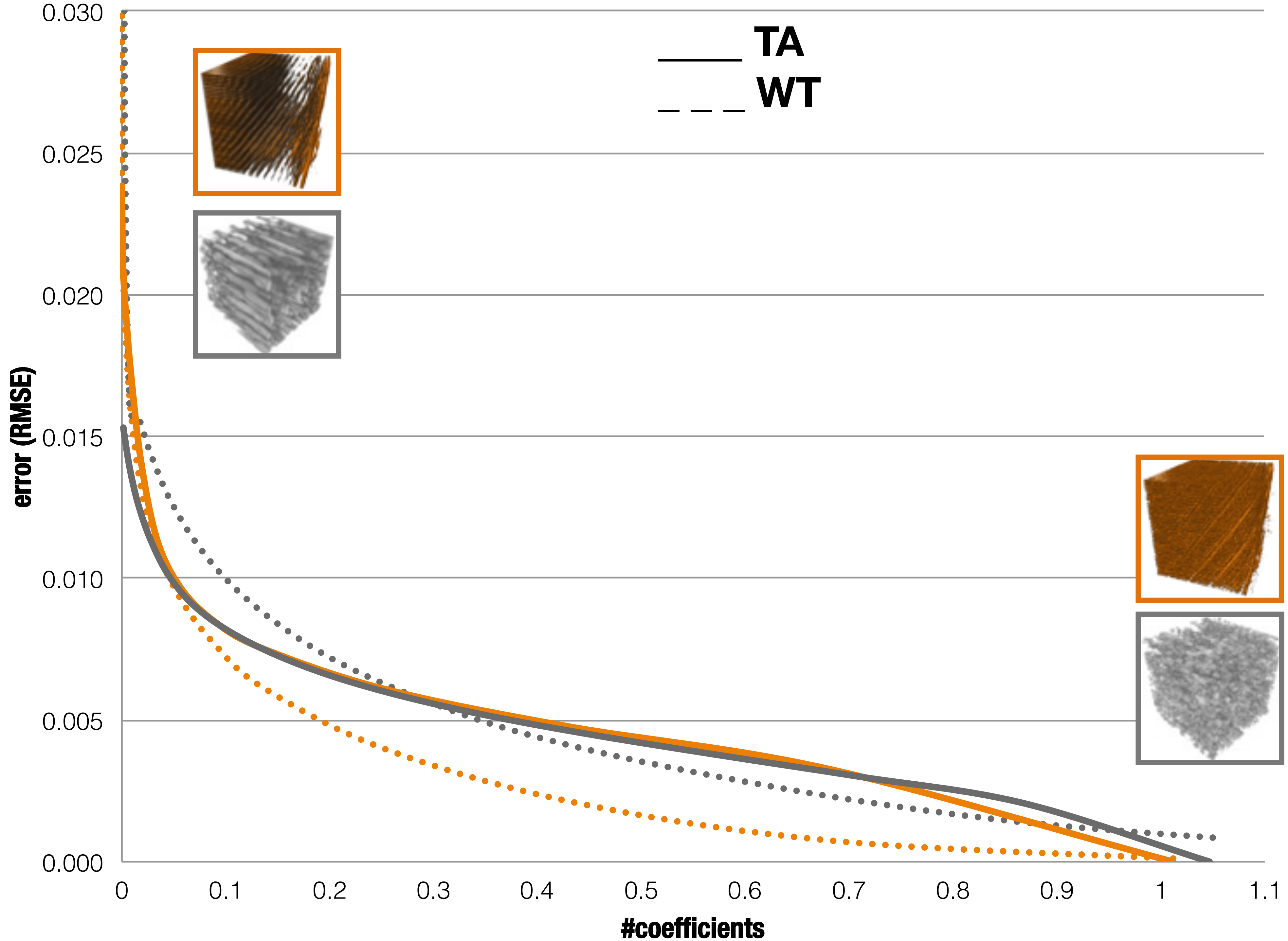
Real Multiscale Dental Growth Structures

[Suter et al., 2010]

original size:
256^3 = 16’777‘216



Reconstruction “Error” vs. Compression



WT vs. TA

Wavelet Transform (WT)

- Recursive decomposition at each scale into coarser resolutions
- Traditional multiresolution:
  - projects signal at different resolutions onto a prescribed bases without knowledge on data
- Axis-aligned data reduction

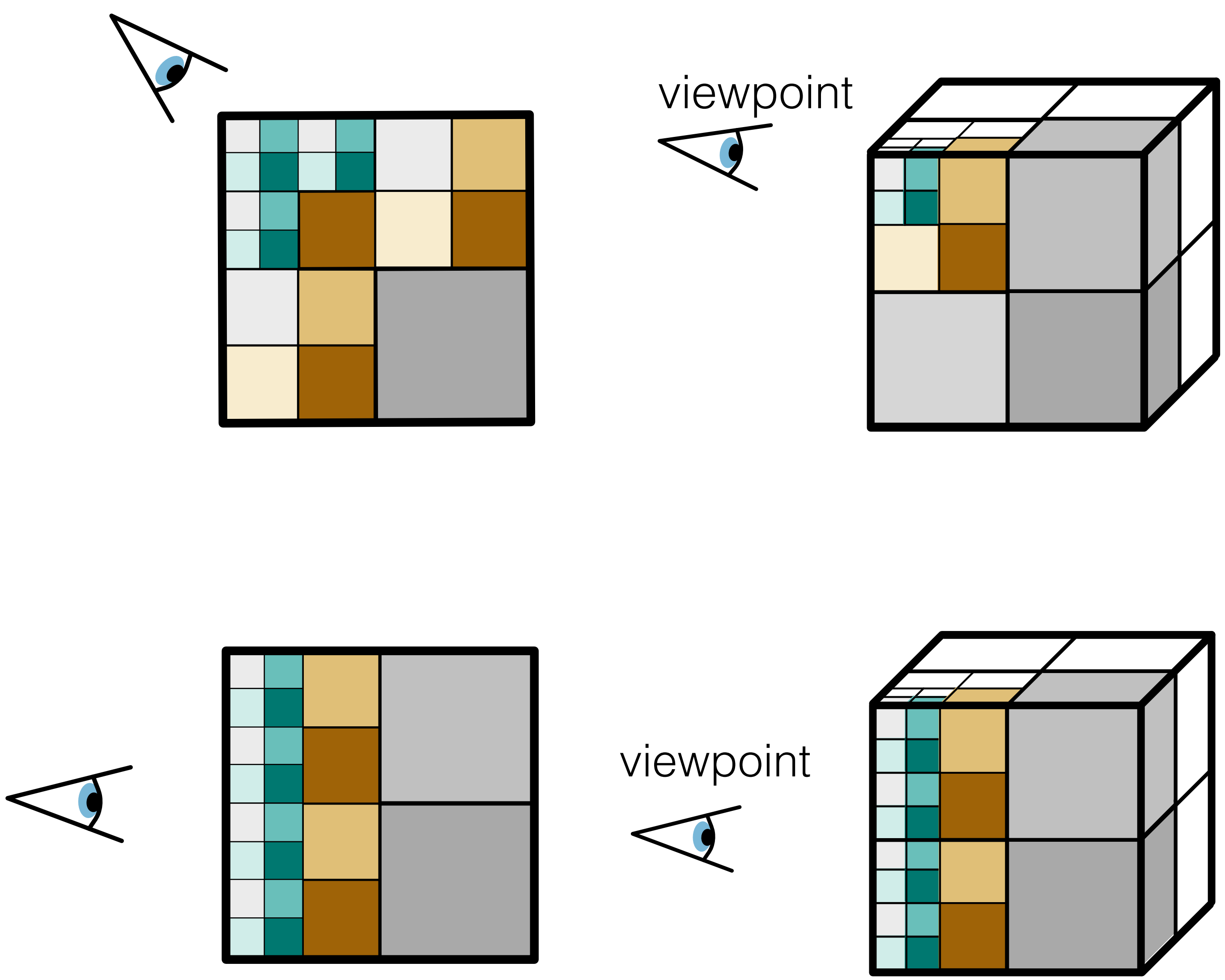
Tensor Approximation (TA)

- Bases are adopted for a given dataset
  - search for major direction/ variability within dataset
- Higher quality images at high data reduction ratios
- Goal: lossy, but keep features
  - analyze and count

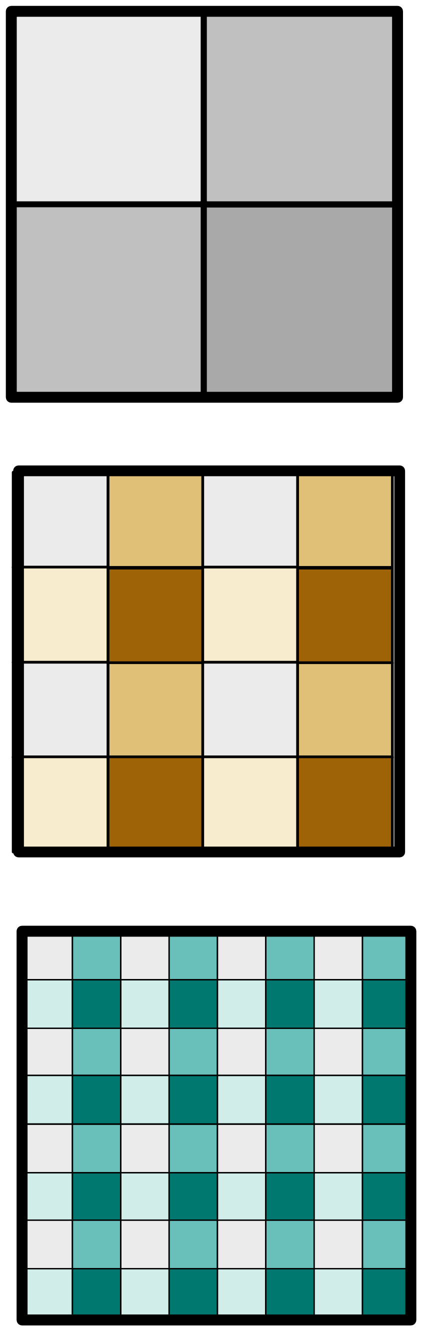
Part 2: Multiresolution TA Hierarchies

Multiresolution Analysis

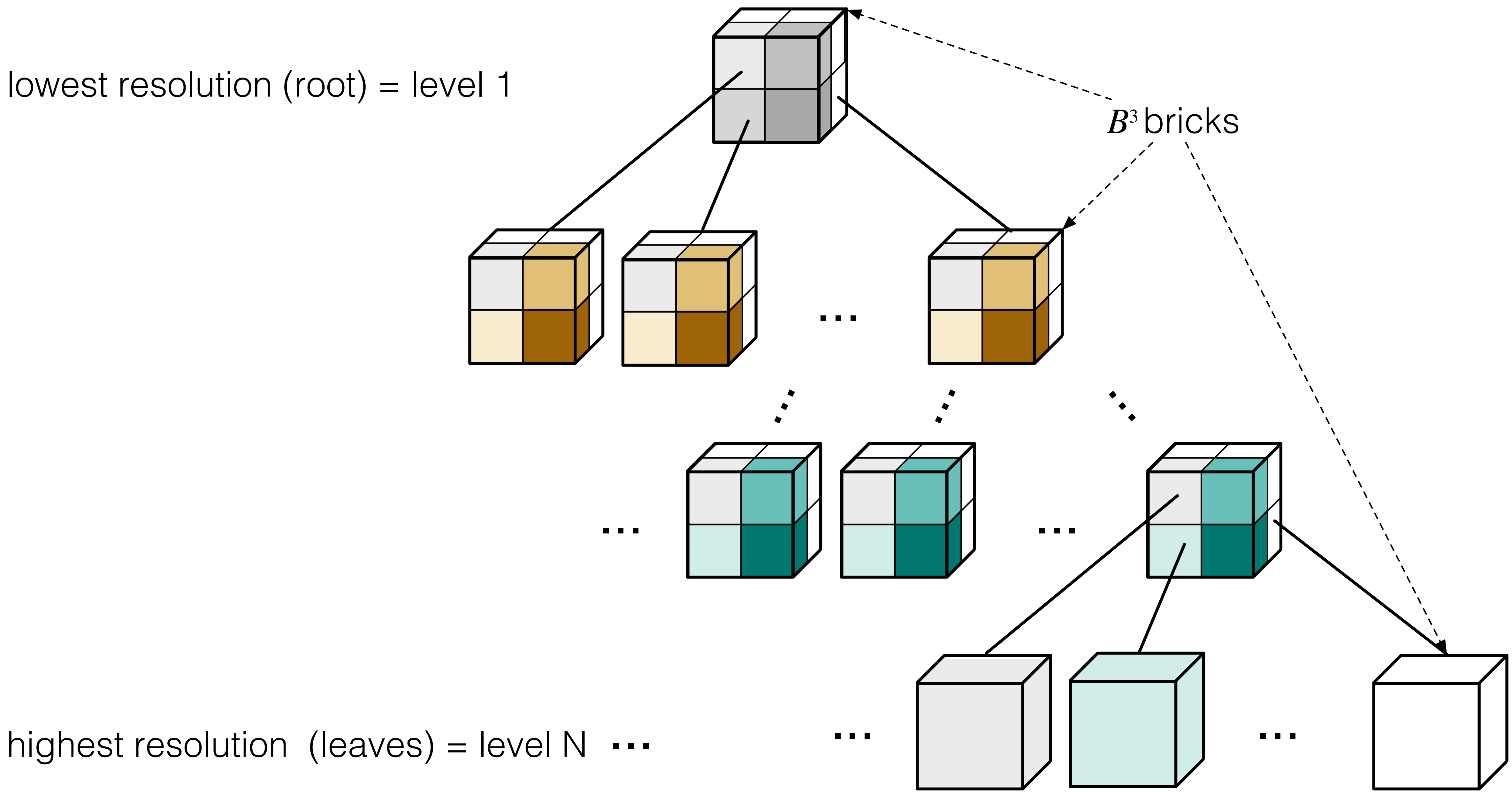
multiresolution visualization



progressive image refinement



Multiresolution Tree Data Structure



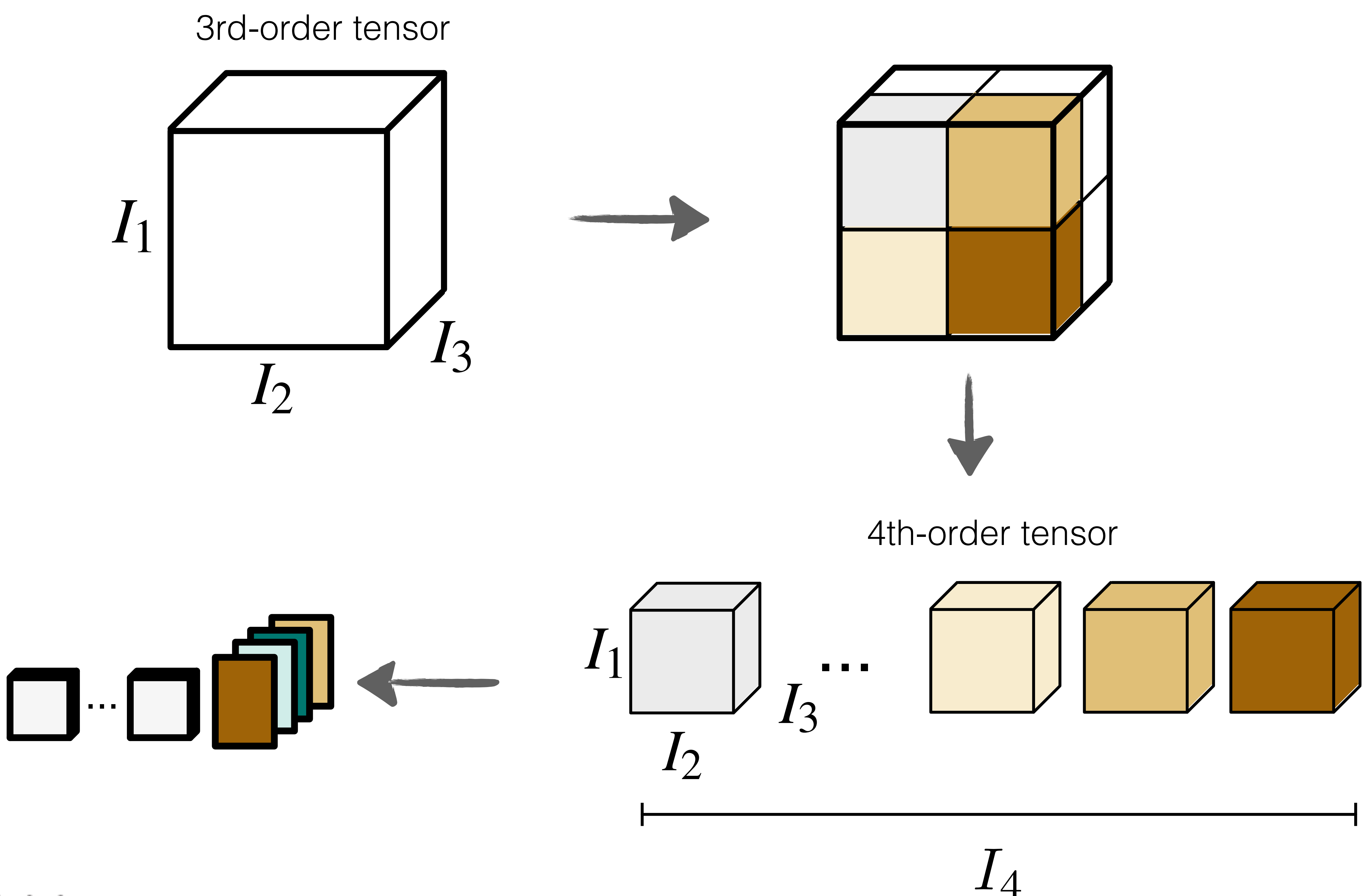
Hierarchical Tucker Model

- Multiresolution analysis
  - significant components at low frequencies
  - less important components at high frequencies
  - high-frequency components have smaller spatial support
  - thus, high-frequency components receive shorter basis vectors
- Why?
  - exploit more redundancy
  - receive smoother borders

Wu et al.. Hierarchical tensor approximation of multidimensional visual data. *IEEE Transactions on Visualization and Computer Graphics* 14(1):186-199, January/February 2008.

Tensor Ensembles

[Wu et al., 2008]



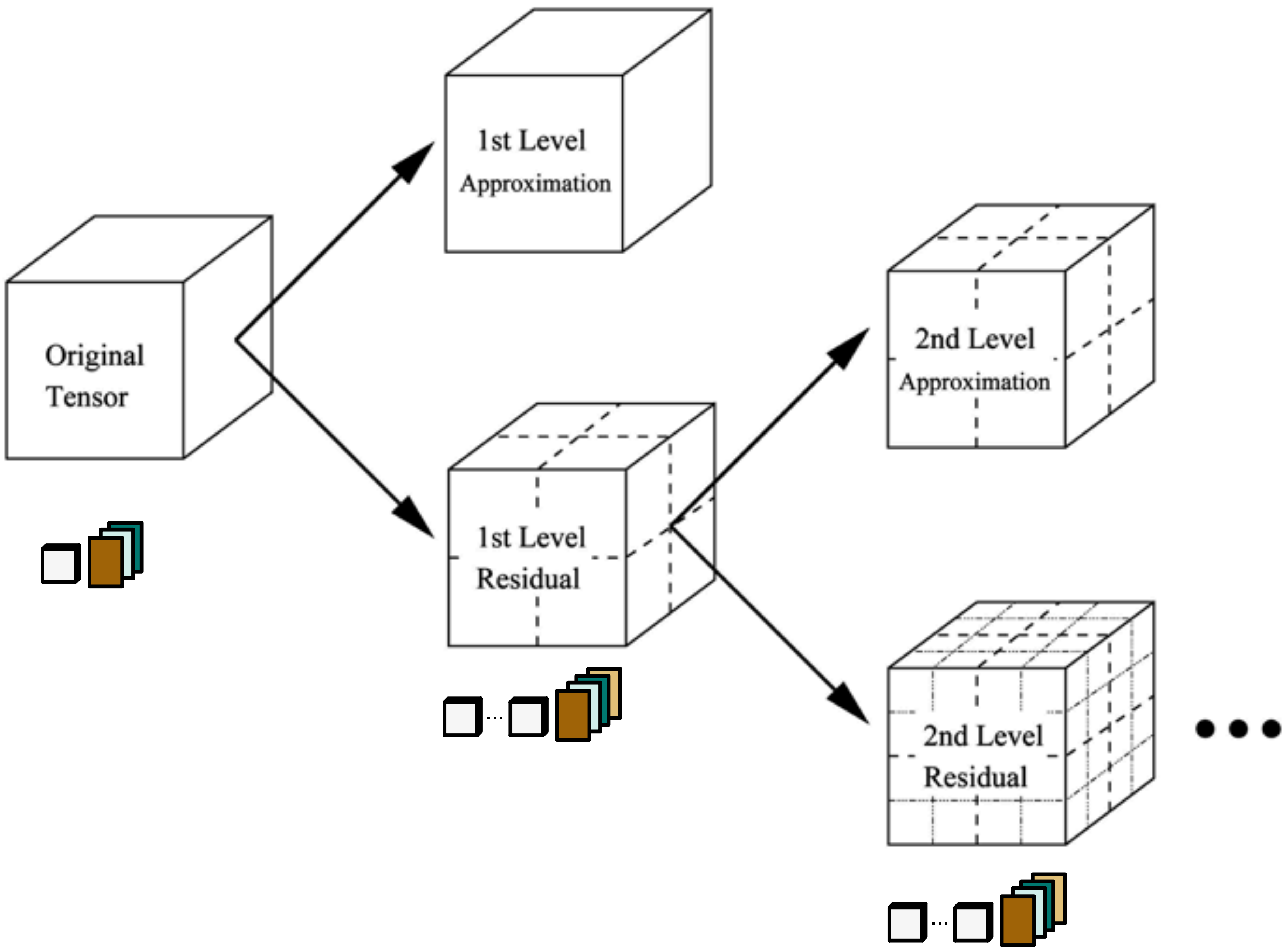
Tensor Ensemble Ranks

[Wu et al., 2008]

- Multilinear rank $(R_1, R_2, \ldots R_N)$ defined per hierarchy:
  - start with $R_n = I_n / 8$ or $I_n / 16$
  - each next hierarchy rank R_n is half of the rank of the previous hierarchy level
  - for example: 32, 16, 8, 4, 2

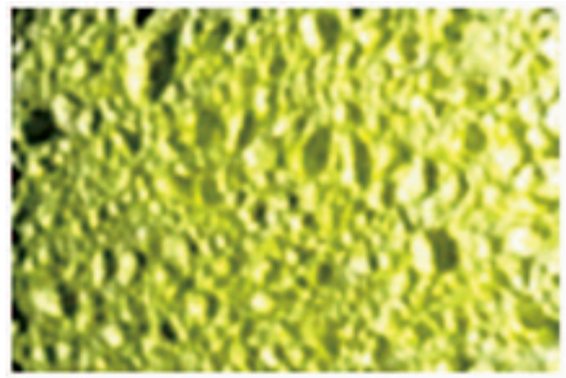
Residual-based Hierarchy

[Wu et al., 2008]

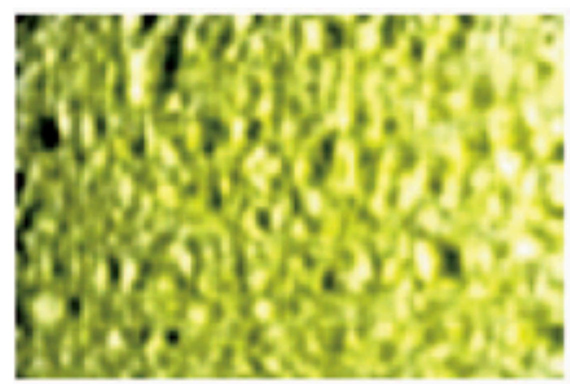
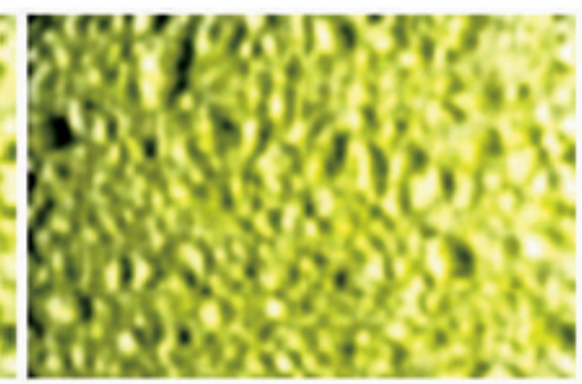


Hierarchical TA and WT on a BTF

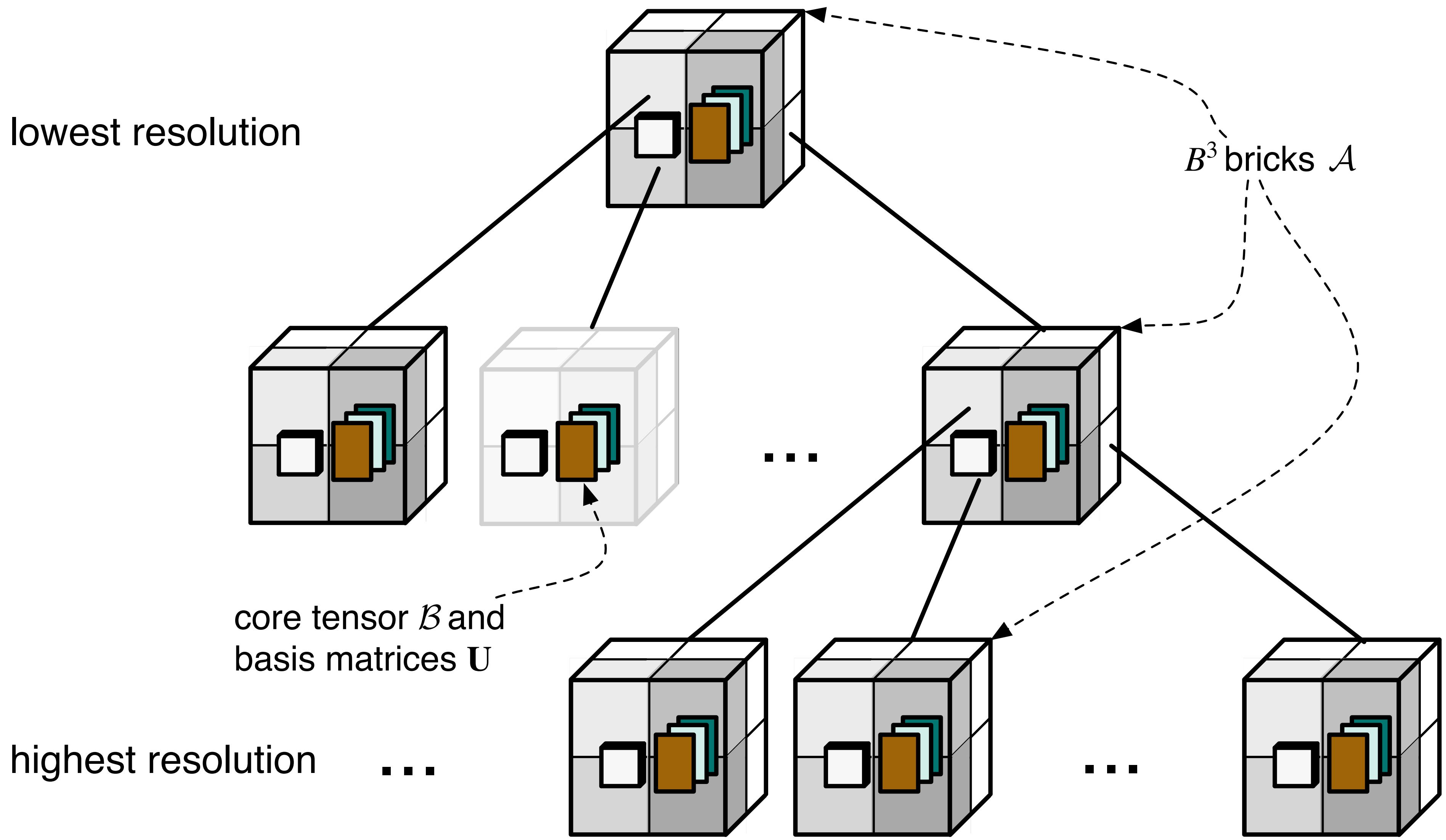
[Wu et al., 2008]



original: sponge BTF
- 45 views
- 60 illuminations
- image: 128x128

biorth. wavelet	**wavelet packet**	**single level TA** [Wang et al., 2005]	**multi level TA**	
				compression ratio: 55
				compression ratio: 3,922

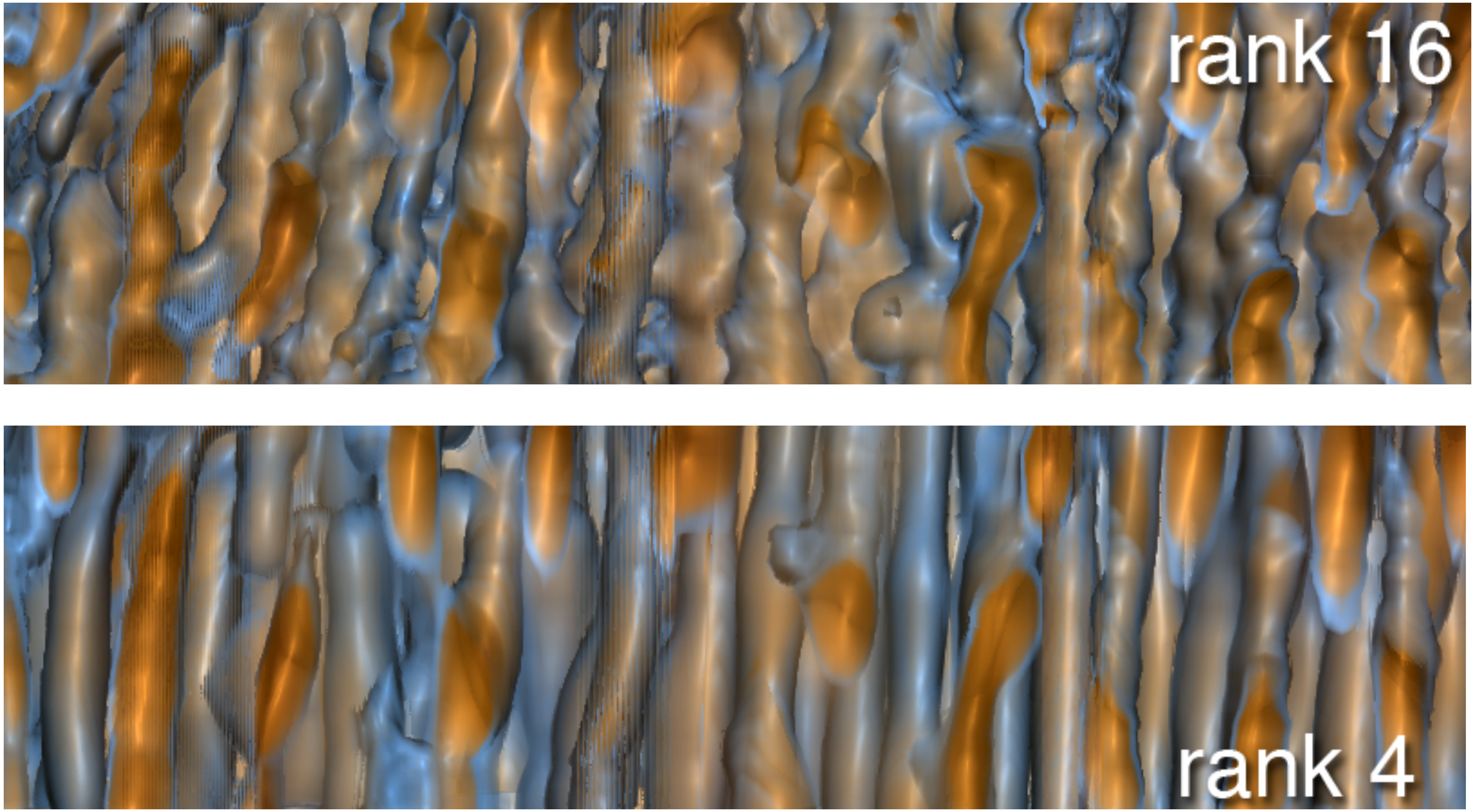
Multiresolution Direct Volume Rendering



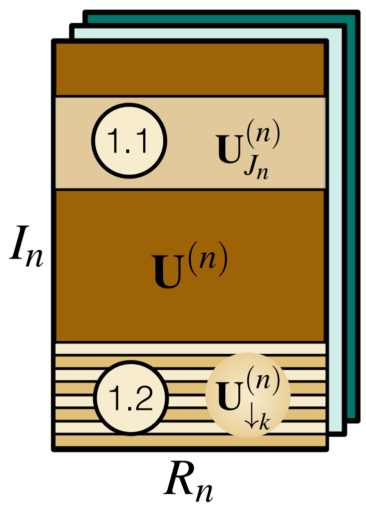
Suter et al.. Interactive multiscale tensor reconstruction for multiresolution volume visualization. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2135–2143, December 2011.

Rank-reducibility and Feature Extraction

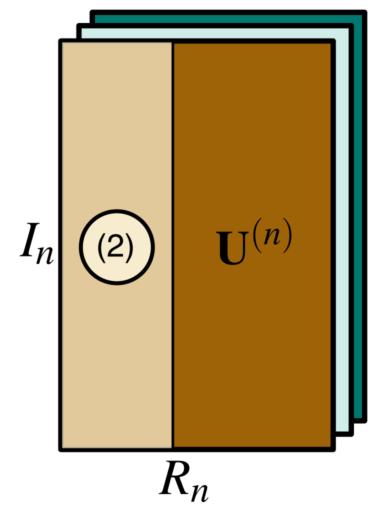
[Suter et al., 2011]



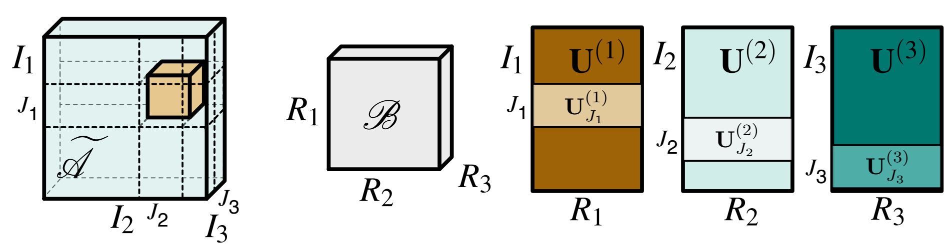
Recap: Tensor Bases and Properties

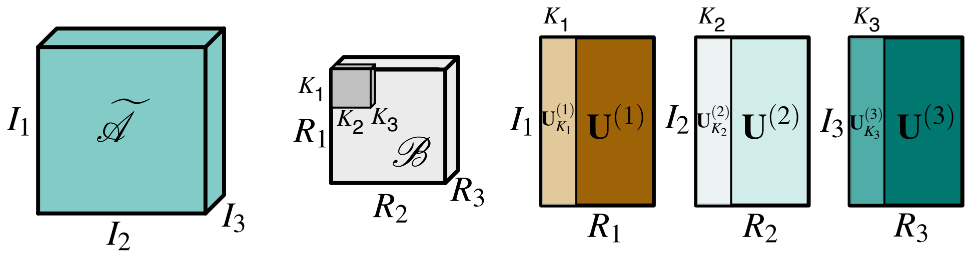


1.1 spatial selection
1.2 spatial subsampling



2 tensor rank truncation





use for state-of-the-art **multiresolution** volume rendering

use for **multiscale** volume feature visualization

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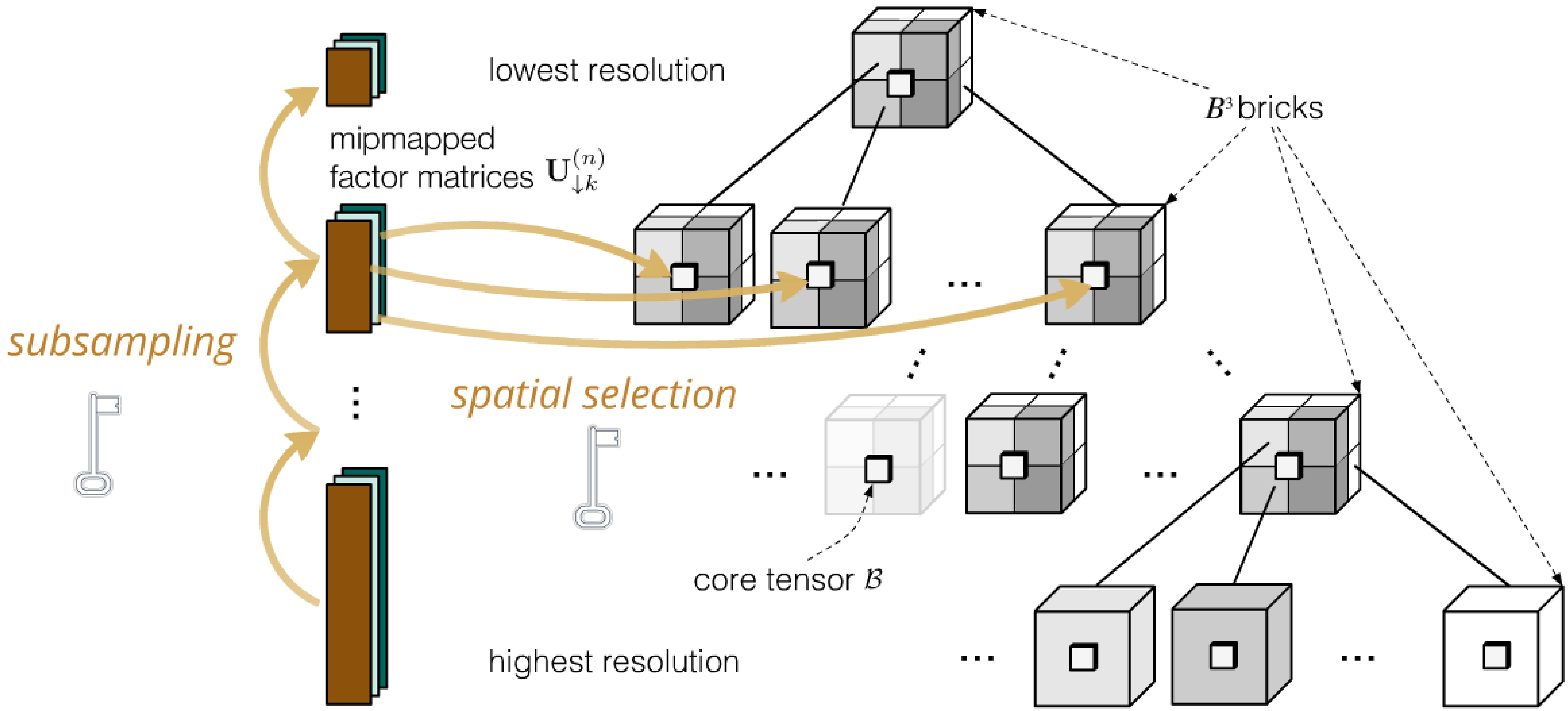
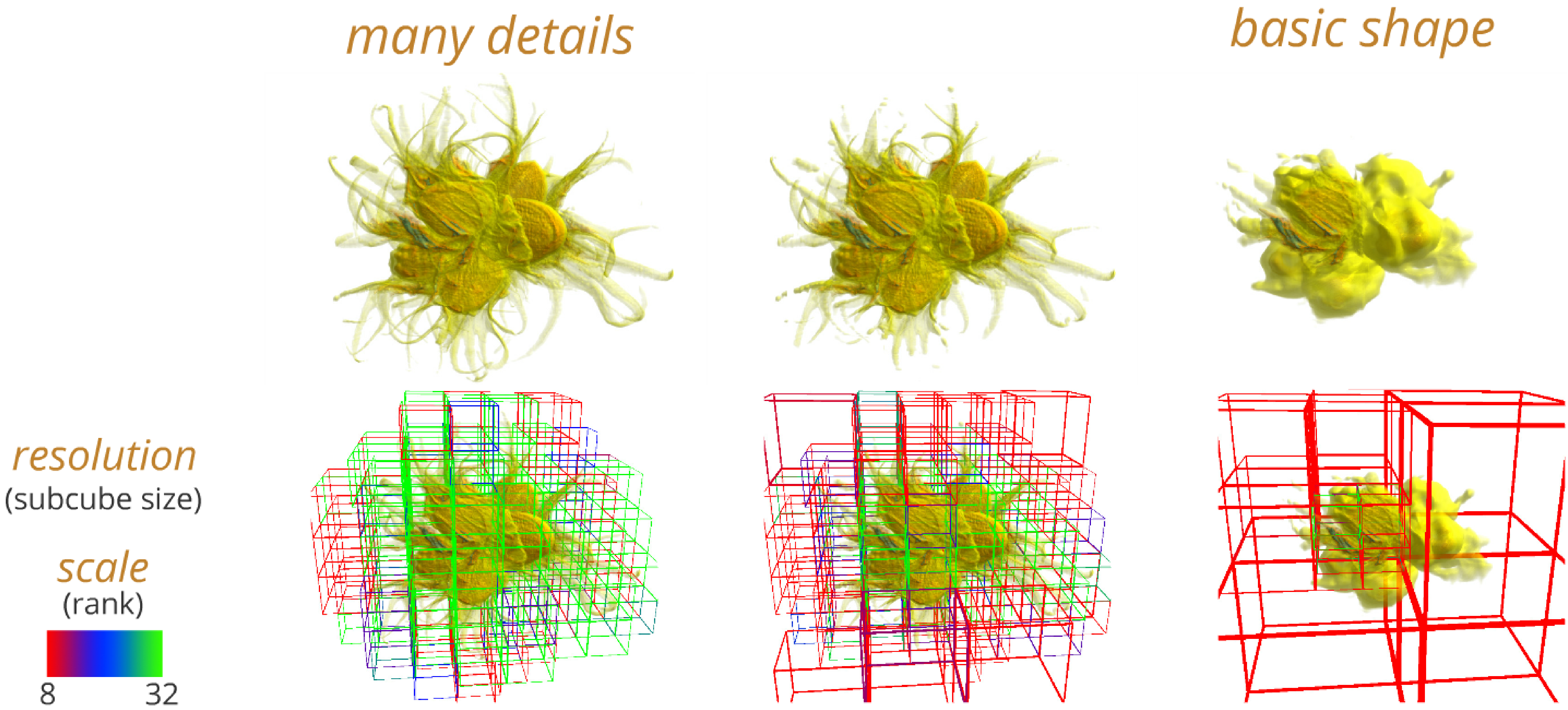
Multiresolution and Multiscale DVR



Suter, Makhinya and Pajarola. TAMRESH: Tensor approximation multiresolution hierarchy for interactive volume visualization. *Computer Graphics Forum*, 2013.

Multiscale and Multiresolution

[Suter et al., 2013]



Storage Costs of TA Hierarchy Models

- Theoretical costs
  - without empty space skipping
  - without pruning/thresholding of coefficients
- Assumptions
  - brick size $B = 64$
  - initial rank $R_{init} = 32$
- **Suter et al., 2011**: $\approx 0.17 \cdot I^3$
- **Suter et al., 2013**: $\approx 192 \cdot I + 0.14 \cdot I^3$

Storage Costs of TA Hierarchy Models

- Core tensors
  - Wu et al., 2008: $O(\log(I) \cdot R^3)$
  - Suter et al., 2013: $O(R^3)$
- Factor matrices
  - Wu et al., 2008: $O(4 \cdot I \cdot R) + O(\frac{I^6}{B^6})$
  - Suter et al., 2013: $O(6 \cdot I \cdot R)$
- Rank
  - Wu et al., 2008: $R = \frac{I}{16}$
  - Suter et al., 2013: $R = \frac{B}{2} = 32$
- Pruning is an important factor for Wu et al., 2008

Quantization of TA Hierarchy Models

- Compact representation coefficients usually floating point numbers
- Quantize coefficients

	factor matrices	core tensors
Wu et al., 2008	8-bit	8…20-bit
Suter et al., 2011	16-bit	8-bit
Suter et al., 2013	32-bit*	8-bit

*no quantization

Conclusion

- Compact data representations in scientific visualization
  - Tucker models
- Multiscale feature extraction
  - tensor rank truncation
- Hierarchical (multiresolution) Tucker models
  - residual-based approach (pruning important) [Wu et al., 2008]
  - simple brick-based multiresolution model [Suter et al., 2011]
  - global bases; multiresolution and multiscalability [Suter et al., 2013]
- Compression via TA

Acknowledgments

- This work was supported by:
  - Forschungskredit of the University of Zürich
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