

Tutorial: Tensor Approximation in
Visualization and Graphics

Scientific Visualization Applications

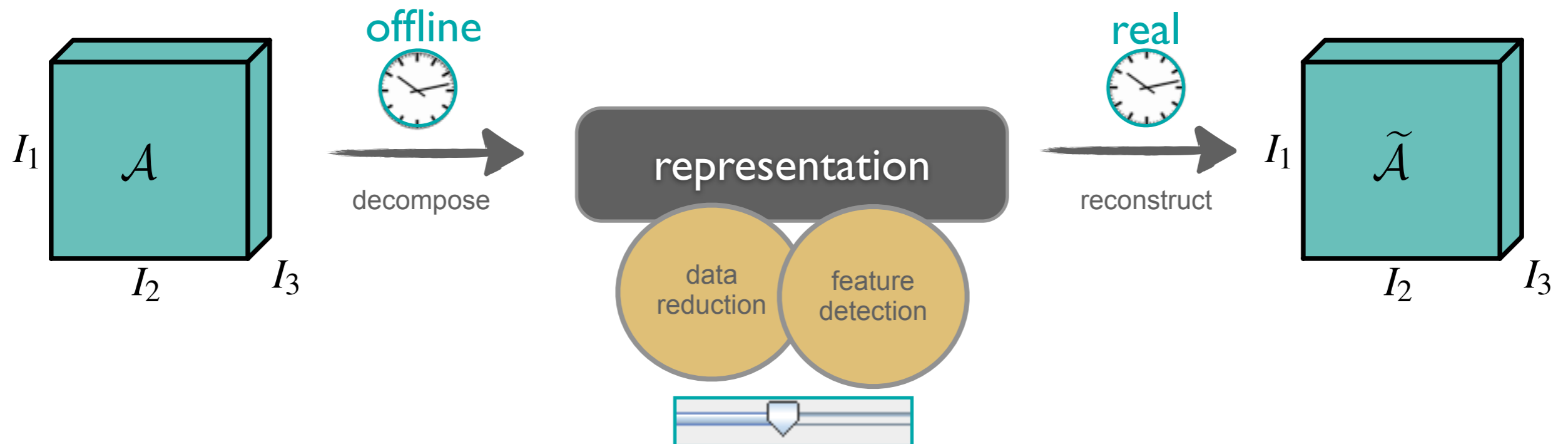
Renato Pajarola, **Susanne K. Suter**, and Roland Ruiters

Outline

- Part 1: Compact data representations compared
 - ▶ wavelets
 - ▶ tensor approximation (Tucker model)
 - ▶ compression and multiscale features
- Part 2: Multiresolution TA Hierarchies

Part 1: Compact Data Representations Compared

Compact Data Representations



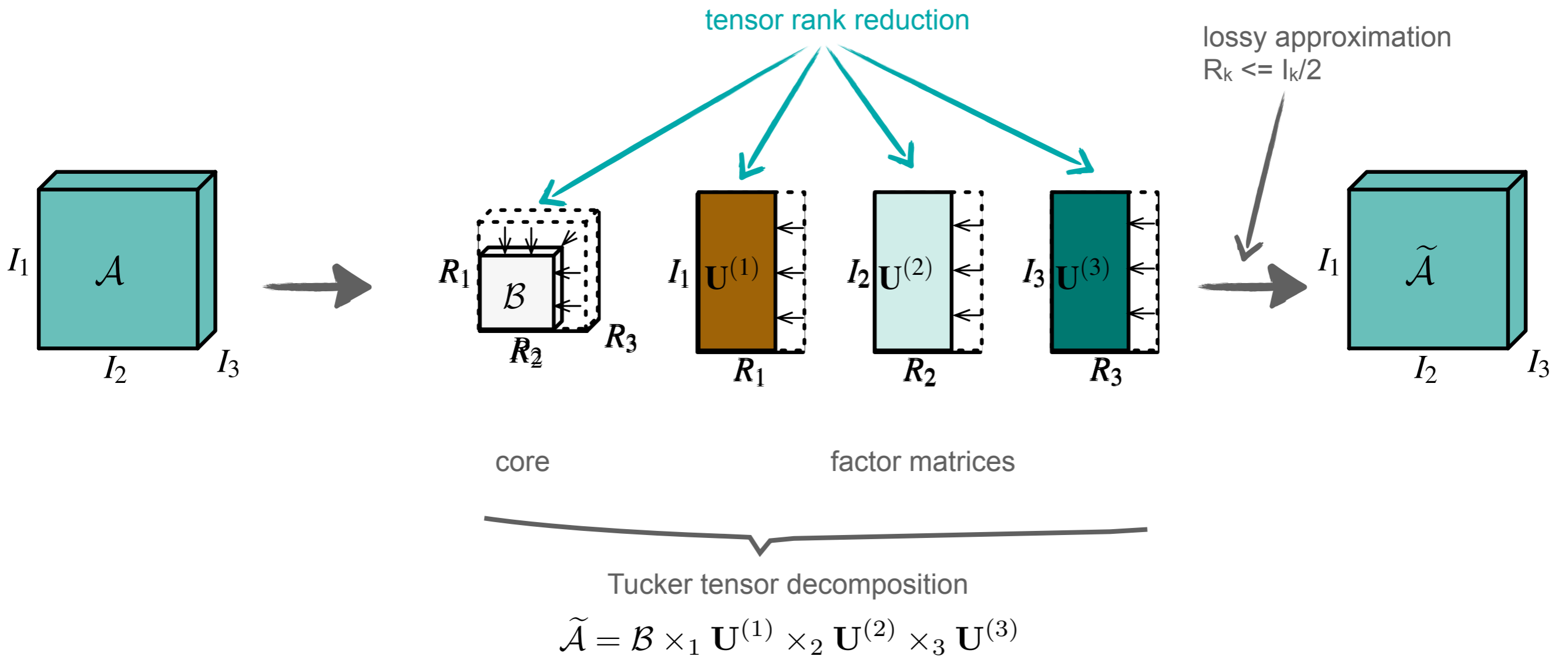
Compact Data Representation Models

- Discrete cosine transform
[Yeo & Liu, 1995]
- Fourier transform
[Chiueh et al, 1997]
- Wavelet transform
[Rodler, 1999; Guthe et al, 2002]
- Vector quantization
[Schneider & Westerman, 2003; Fout & Ma, 2007; Parys & Knittel, 2009]
- Tensor approximation
[Tsai & Shih, 2006+2012; Wang et al., 2005; Wu et al, 2008; Suter et al., 2010+2011+2013]
- For details go to EG13 STAR on “A Survey of Compressed GPU Direct Volume Rendering”
(Thursday, 11:00-12:40 in Room C.1)

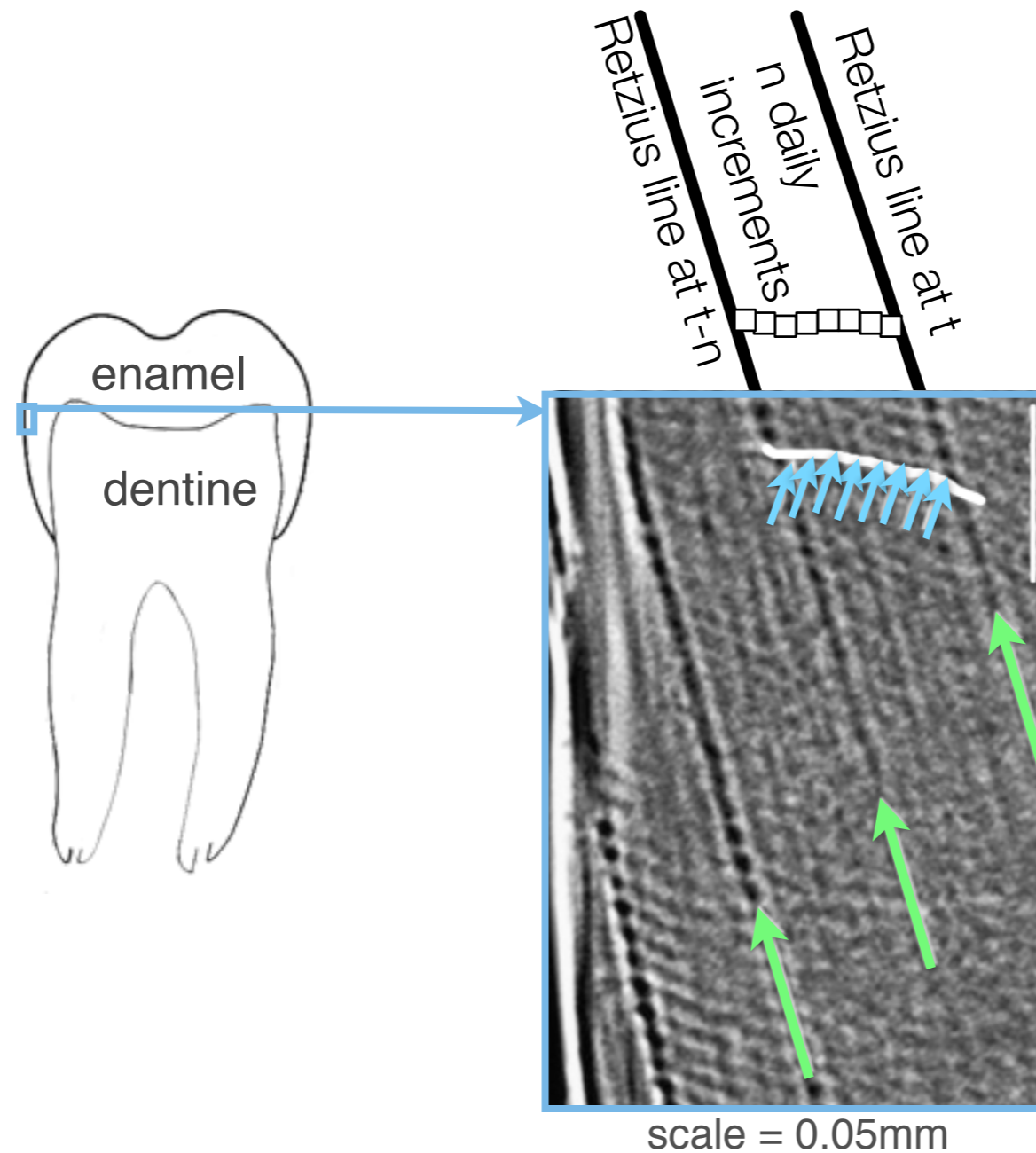
Feature Extraction

- Typically done in multiresolution analysis
 - ▶ significant components at low frequencies
 - ▶ less important components at high frequencies
- Features at multiple spatial scales
- Multiscale feature extraction can be achieved through tensor rank truncation

Tensor Rank Truncation (Tucker Model)



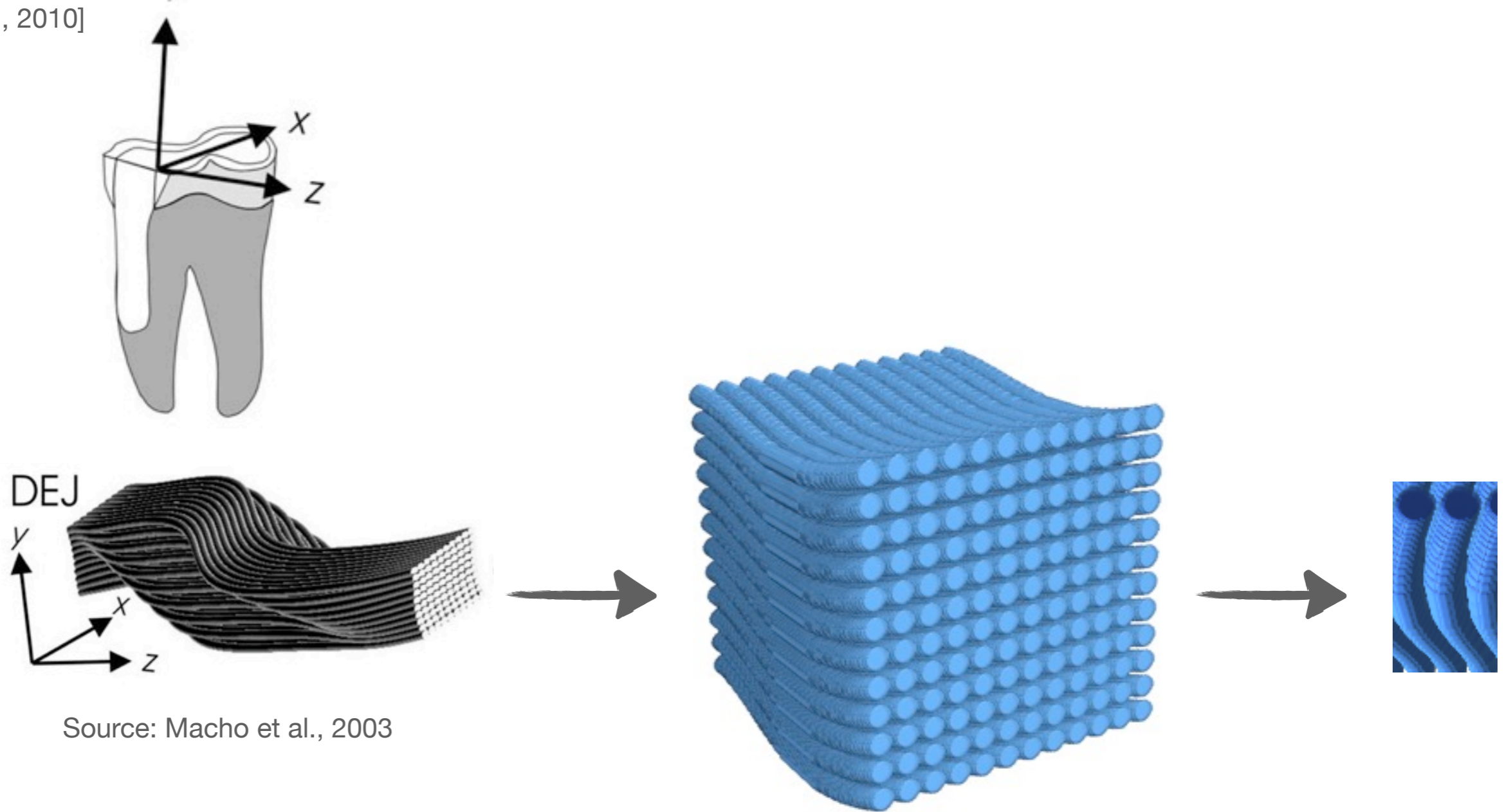
Application: Multiscale Dental Growth Pattern



Suter, Zollikofer and Pajarola. Application of tensor approximation to multiscale volume feature representations. In *Proceedings Vision, Modeling and Visualization*, pages 203–210, 2010.

Synthetic 3D Dental Growth Structures

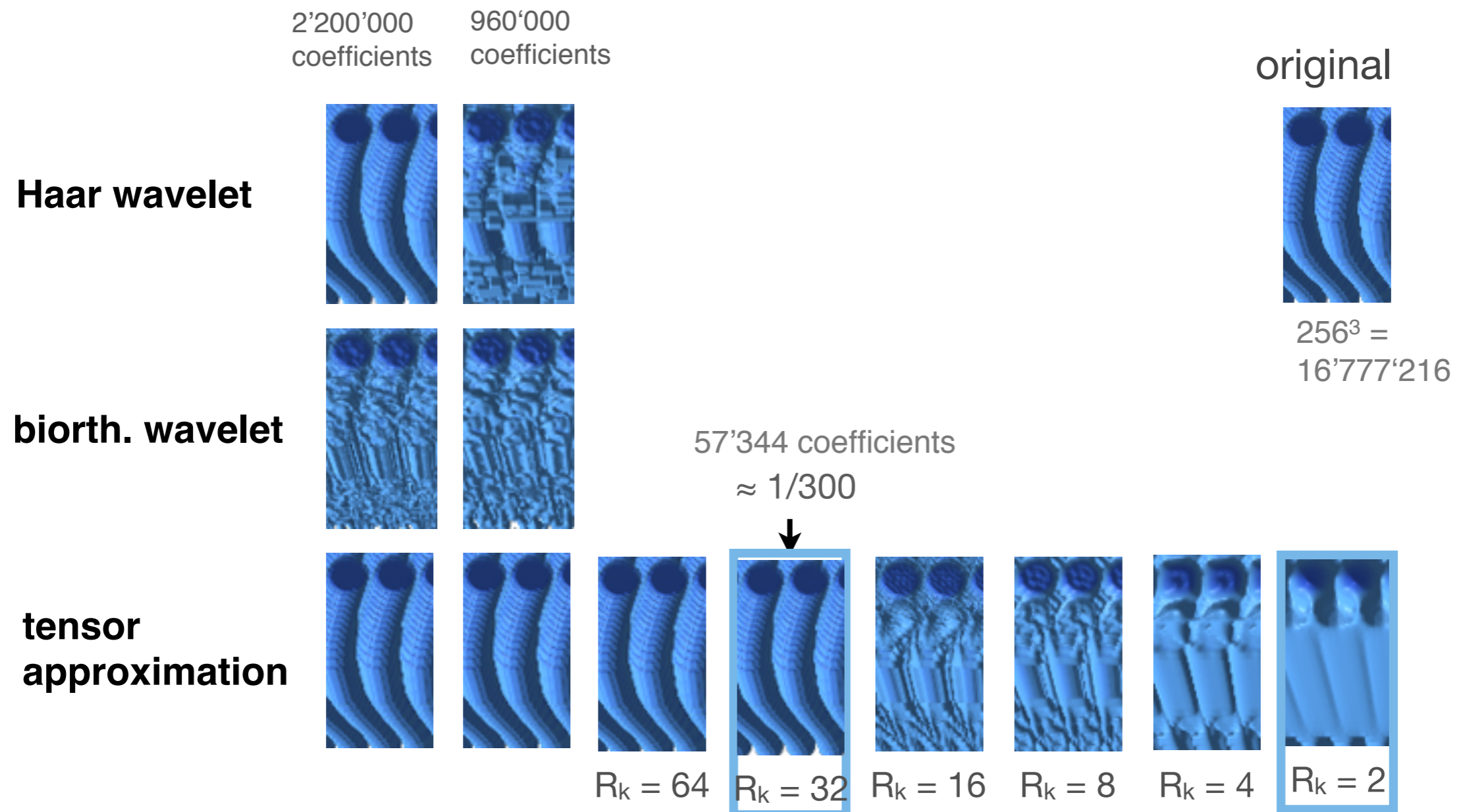
[Suter et al., 2010]



Source: Macho et al., 2003

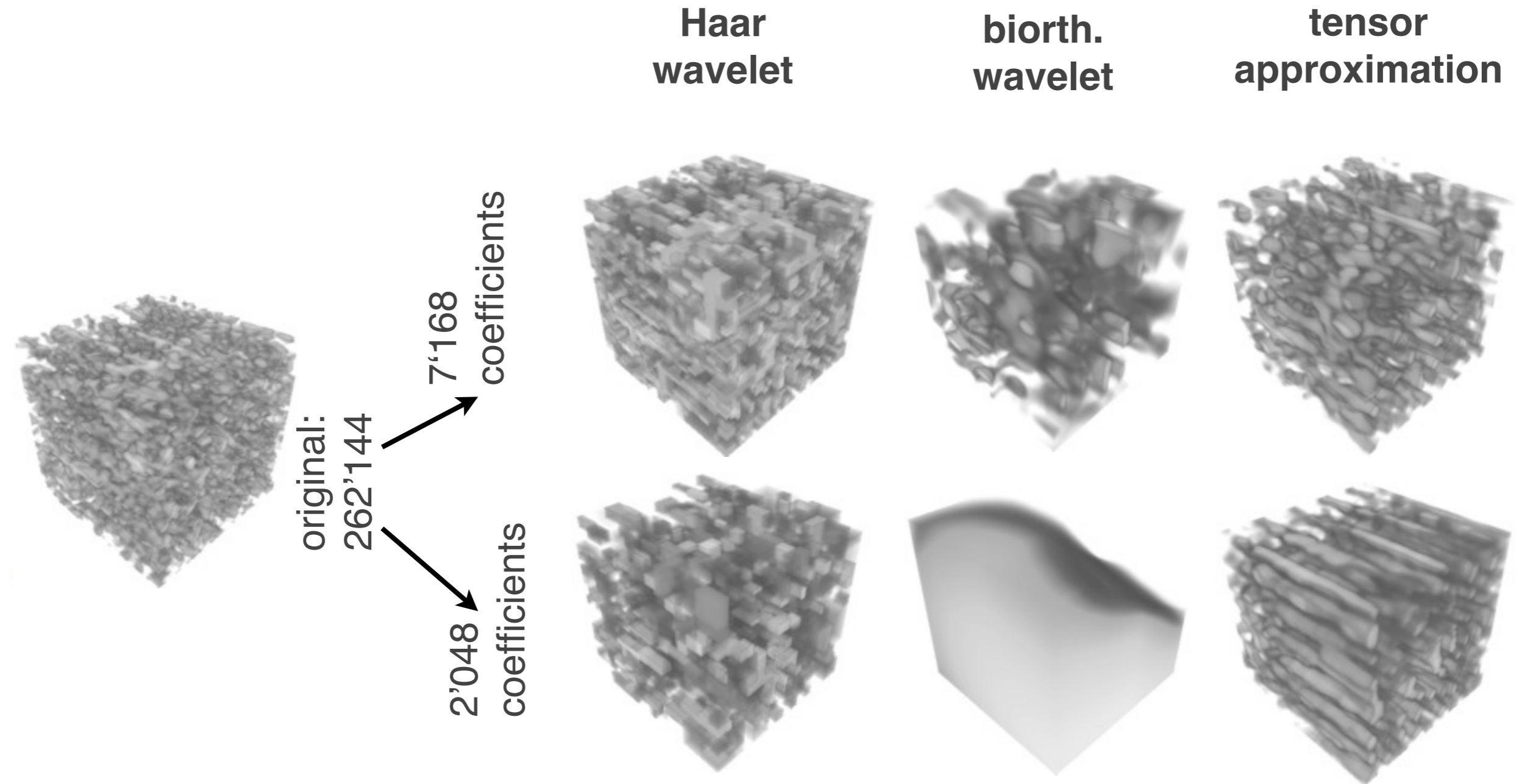
Non-axis-aligned Synthetic Features

[Suter et al., 2010]



Real Dental Growth Structures

[Suter et al., 2010]

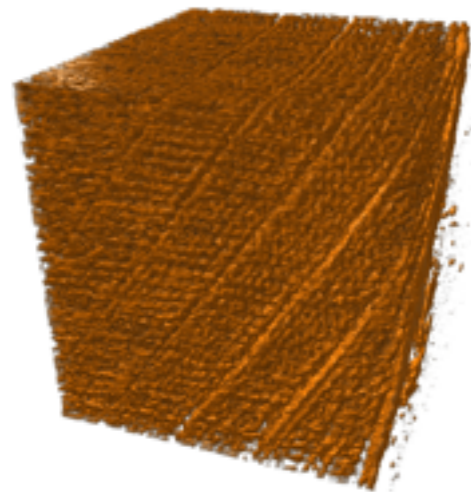


Real Multiscale Dental Growth Structures

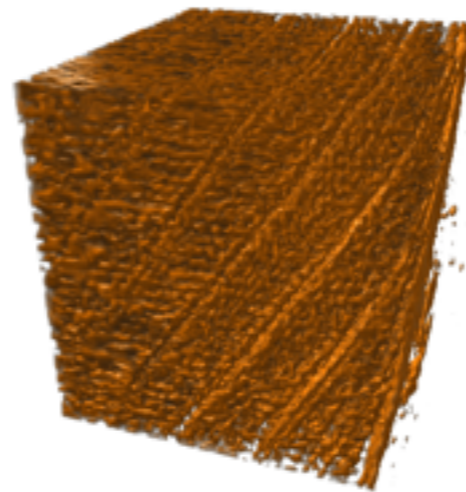
[Suter et al., 2010]

original size:
 $256^3 = 16'777'216$

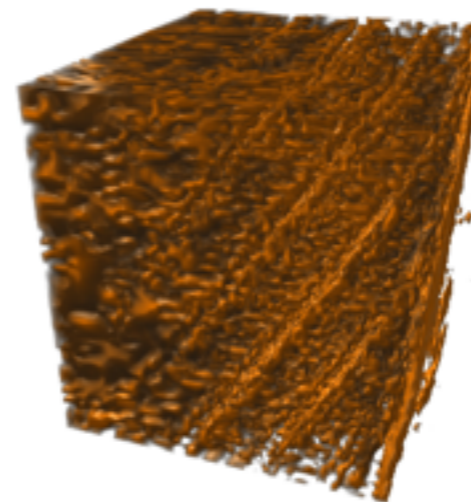
WT



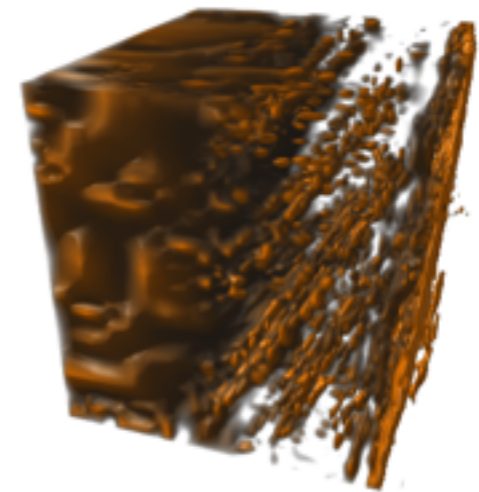
2'200'000 coeff.



310'000 coeff.

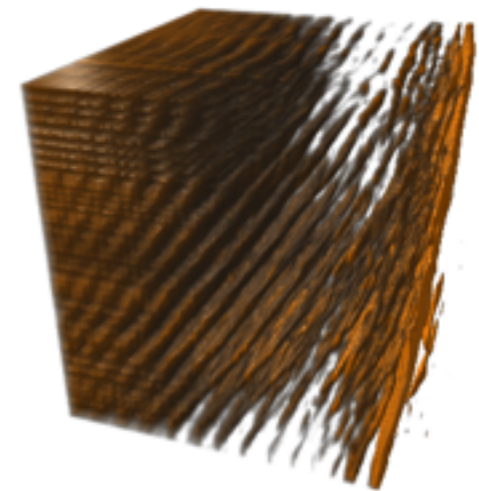
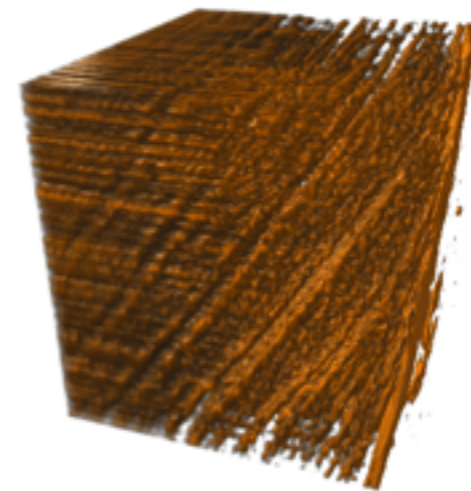
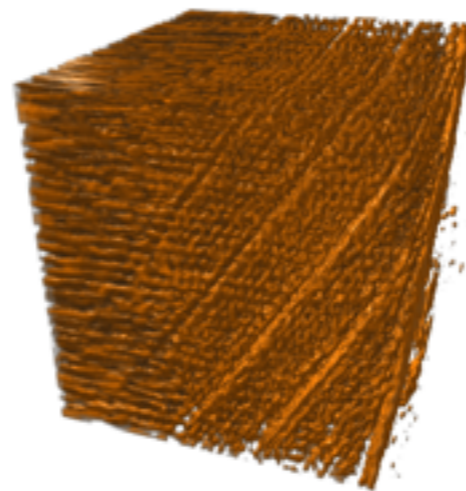
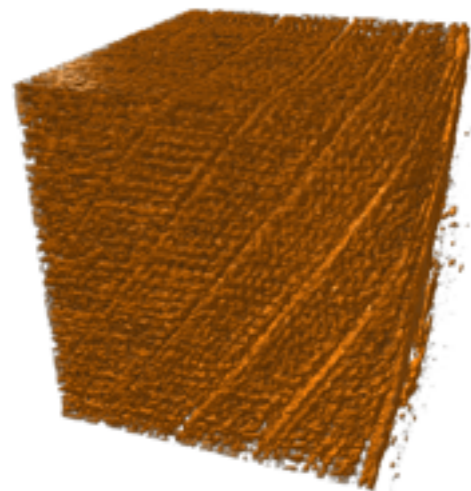


57'500 coeff.

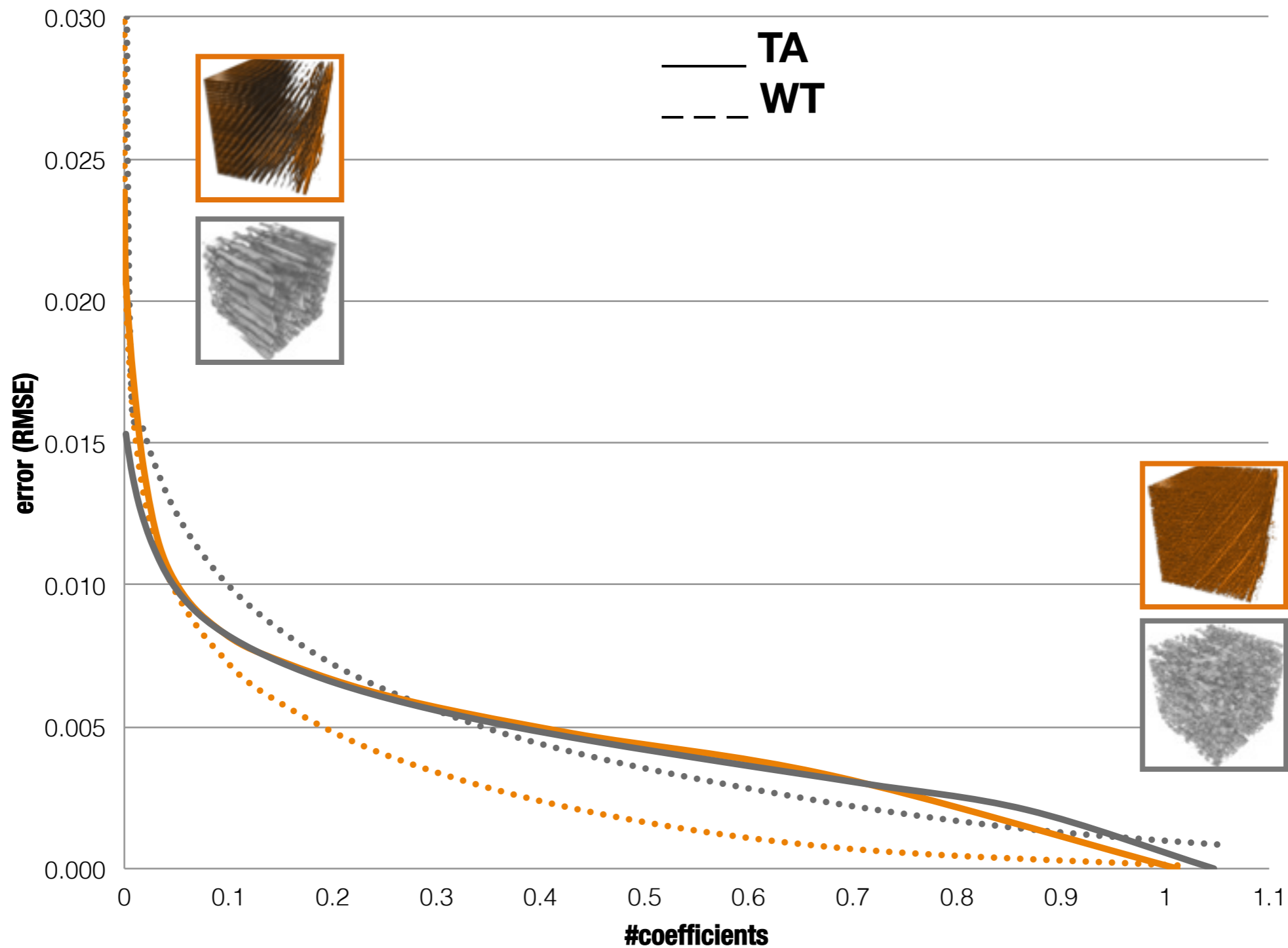


16'500 coeff.

TA



Reconstruction “Error” vs. Compression



WT vs. TA

Wavelet Transform (WT)

- Recursive decomposition at each scale into coarser resolutions
- Traditional multiresolution:
 - ▶ projects signal at different resolutions onto a prescribed bases without knowledge on data
- Axis-aligned data reduction

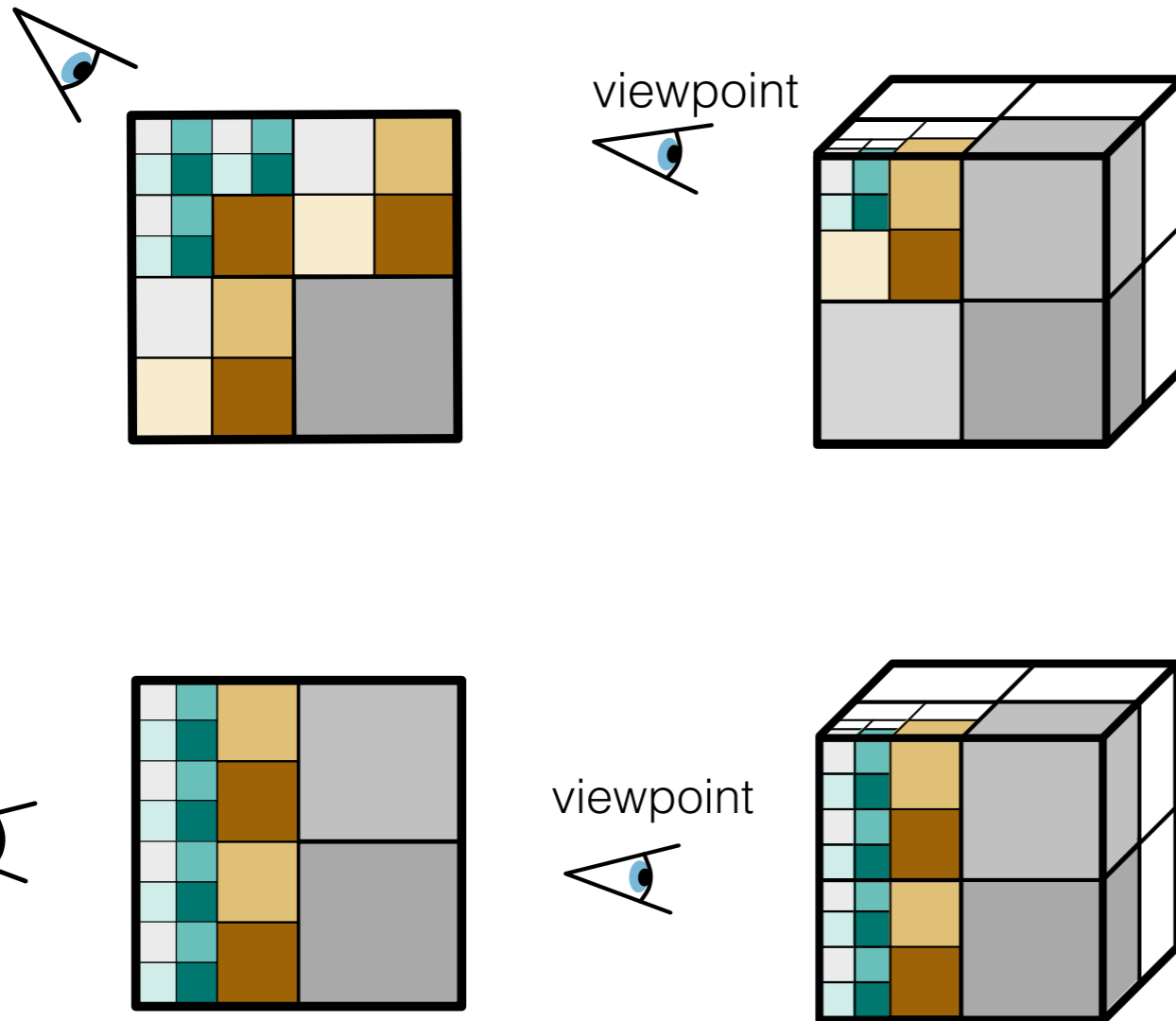
Tensor Approximation (TA)

- Bases are adopted for a given dataset
 - ▶ search for major direction/variability within dataset
- Higher quality images at high data reduction ratios
- Goal: lossy, but keep features
 - ▶ analyze and count

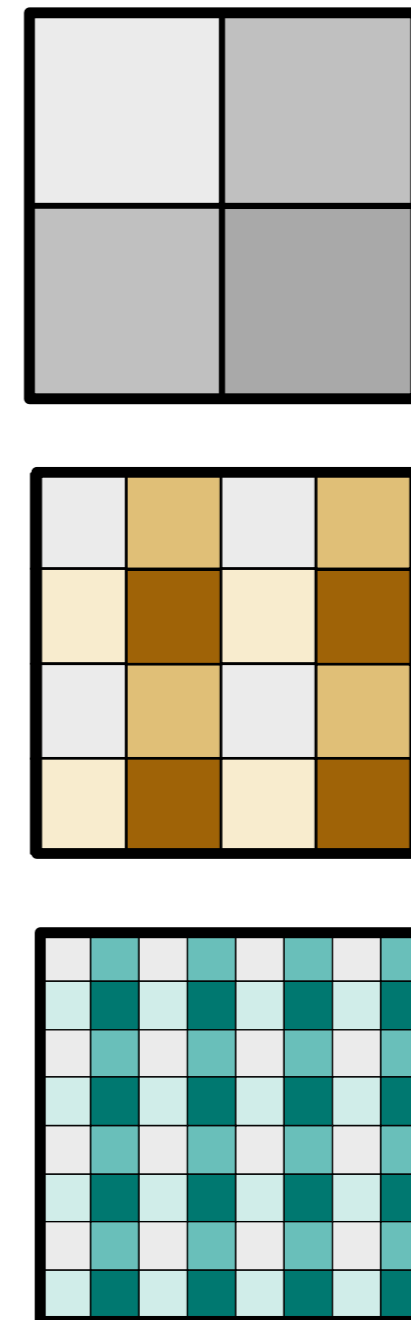
Part 2: Multiresolution TA Hierarchies

Multiresolution Analysis

multiresolution visualization

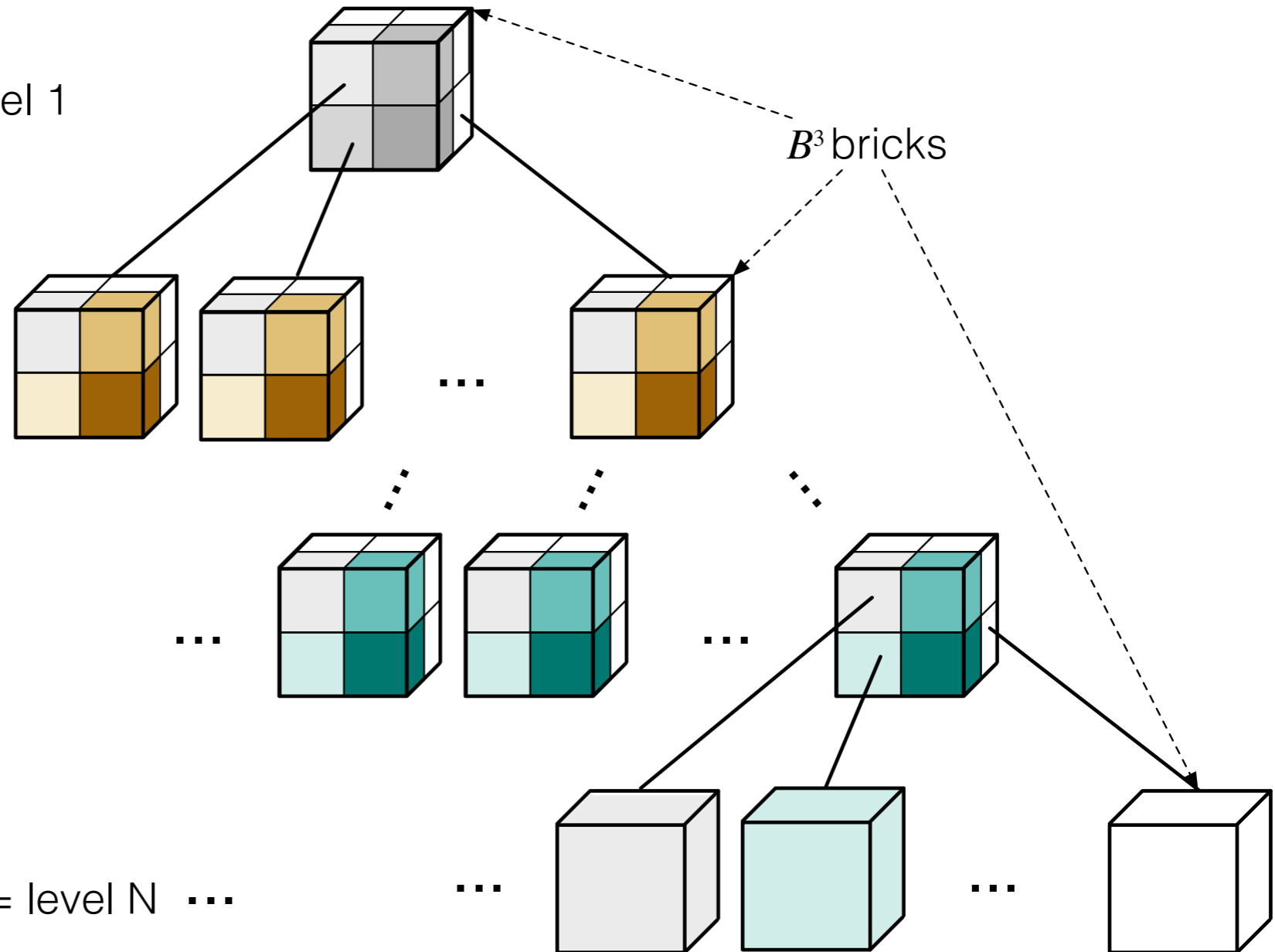


progressive image refinement



Multiresolution Tree Data Structure

lowest resolution (root) = level 1



highest resolution (leaves) = level N ...

Hierarchical Tucker Model

- Multiresolution analysis
 - ▶ significant components at low frequencies
 - ▶ less important components at high frequencies
 - ▶ high-frequency components have smaller spatial support
 - ▶ thus, high-frequency components receive shorter basis vectors
- Why?
 - ▶ exploit more redundancy
 - ▶ receive smoother borders

Wu et al.. Hierarchical tensor approximation of multidimensional visual data. *IEEE Transactions on Visualization and Computer Graphics* 14(1):186-199, January/February 2008.

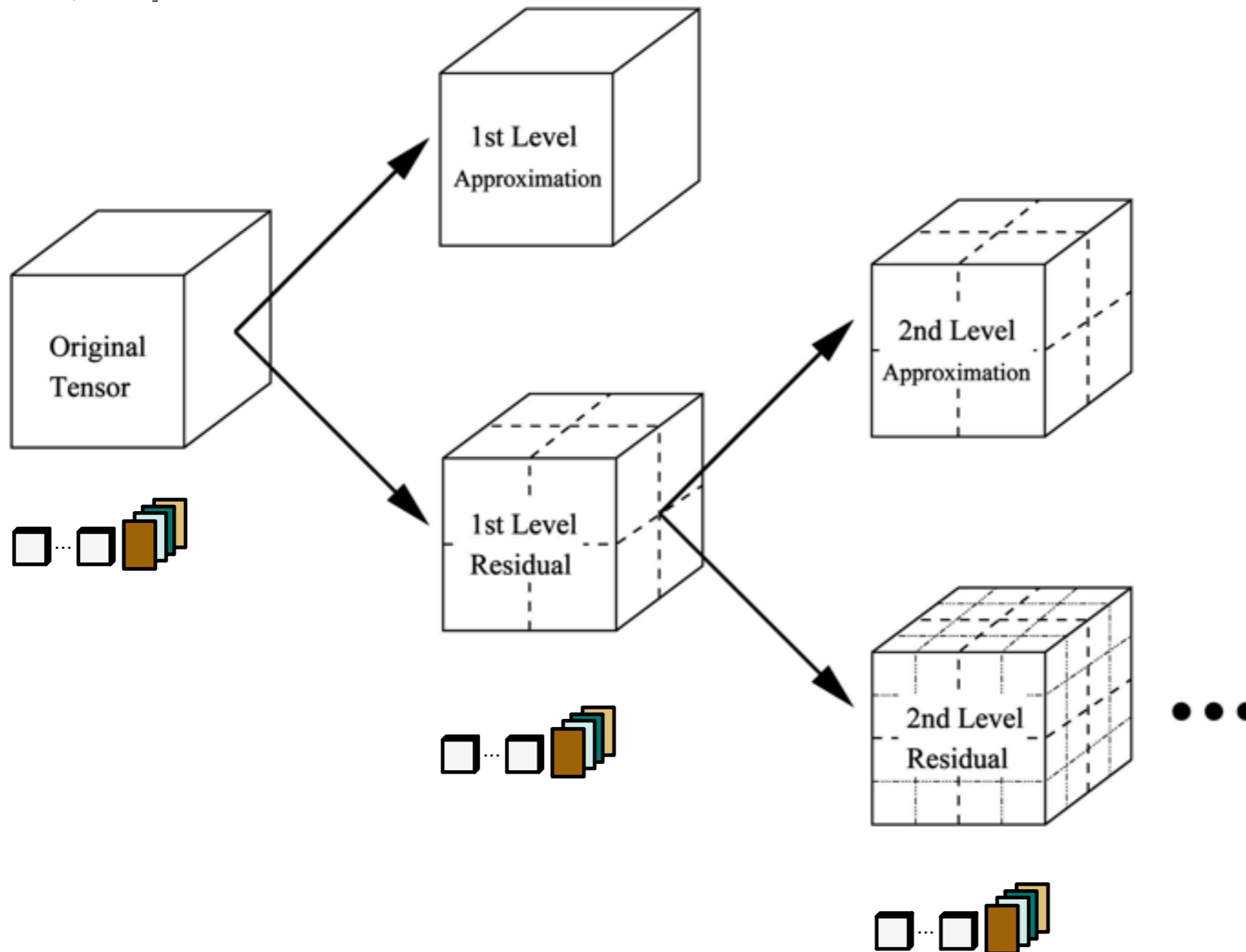
Tensor Ensembles

[Wu et al., 2008]

- At each hierarchy level:
 - ▶ **tensor ensemble:** Put N subtensors into an $(N+1)^{\text{th}}$ -order tensor
 - ▶ perform TA collectively on the tensor ensemble
 - ▶ receive: One set of $(N+1)$ factor matrices and a core tensor
- Multilinear rank (R_1, R_2, \dots, R_N) defined per hierarchy:
 - ▶ start with R_n
 - ▶ each next hierarchy rank R_n is half of the rank of the previous hierarchy level
 - ▶ for example: 32, 16, 8, 4, 2

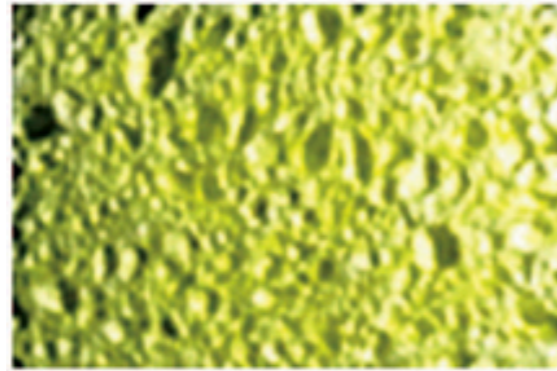
Residual-based Hierarchy

[Wu et al., 2008]



Hierarchical TA and WT on a BTF

[Wu et al., 2008]



original: sponge BTF

- 45 views
- 60 illuminations
- image: 128x128

biorth. wavelet

wavelet packet

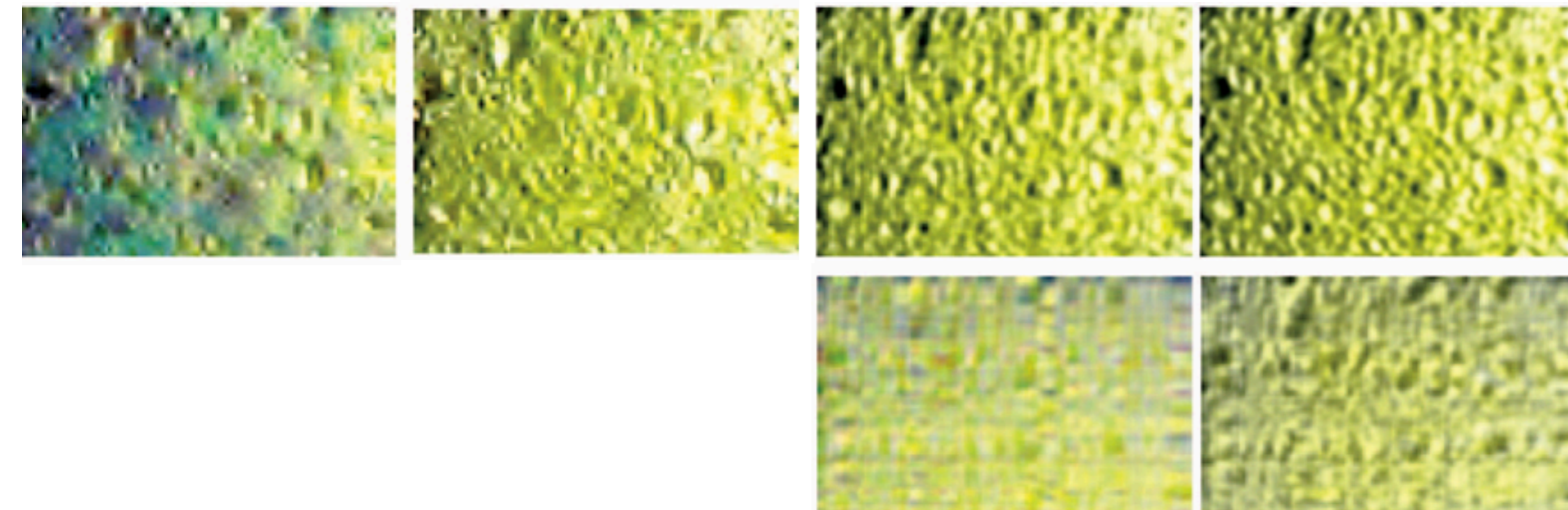
single level TA

multi level TA

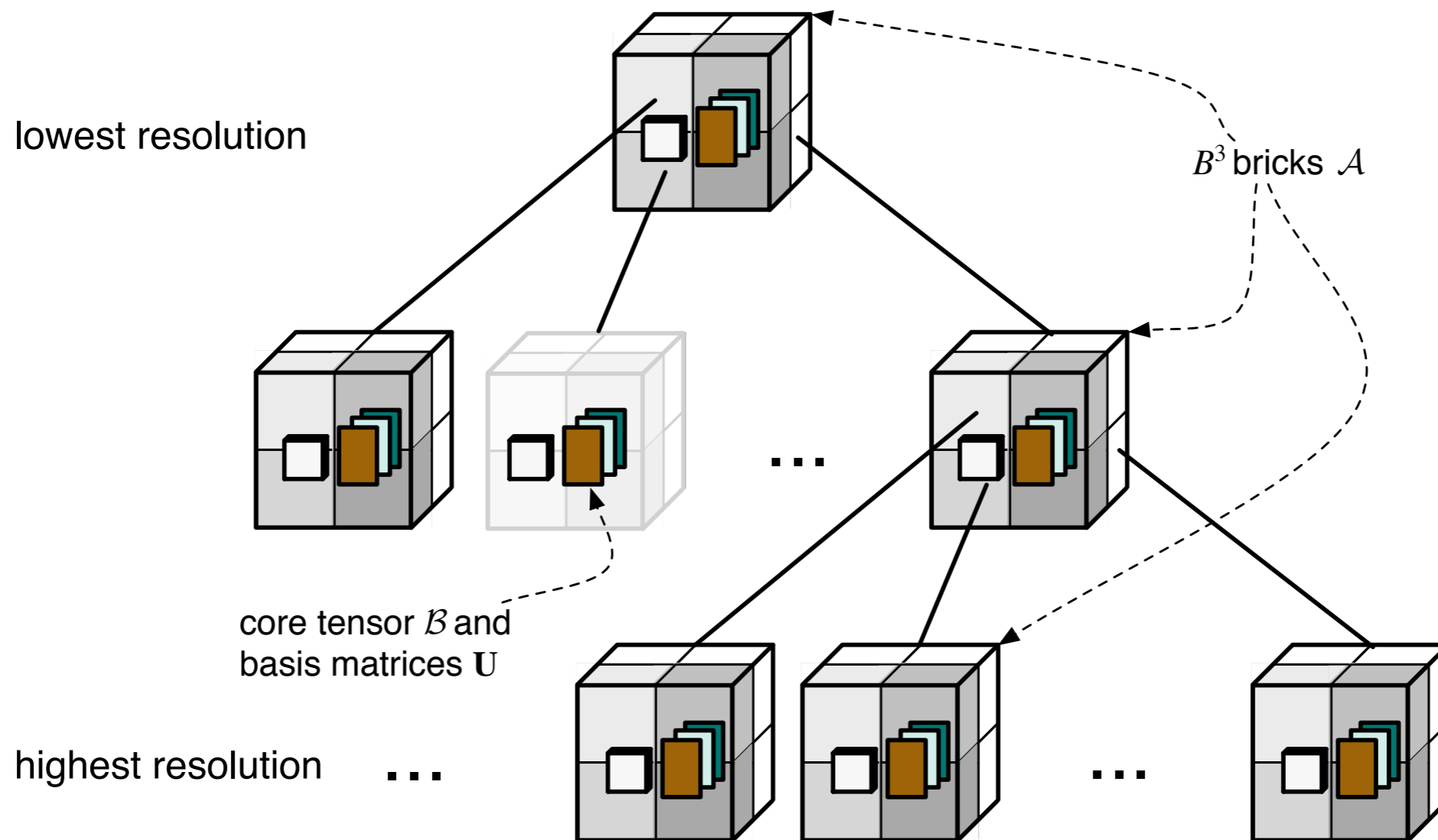
[Wang et al., 2005]

compression
ratio: 55

compression
ratio: 3,922



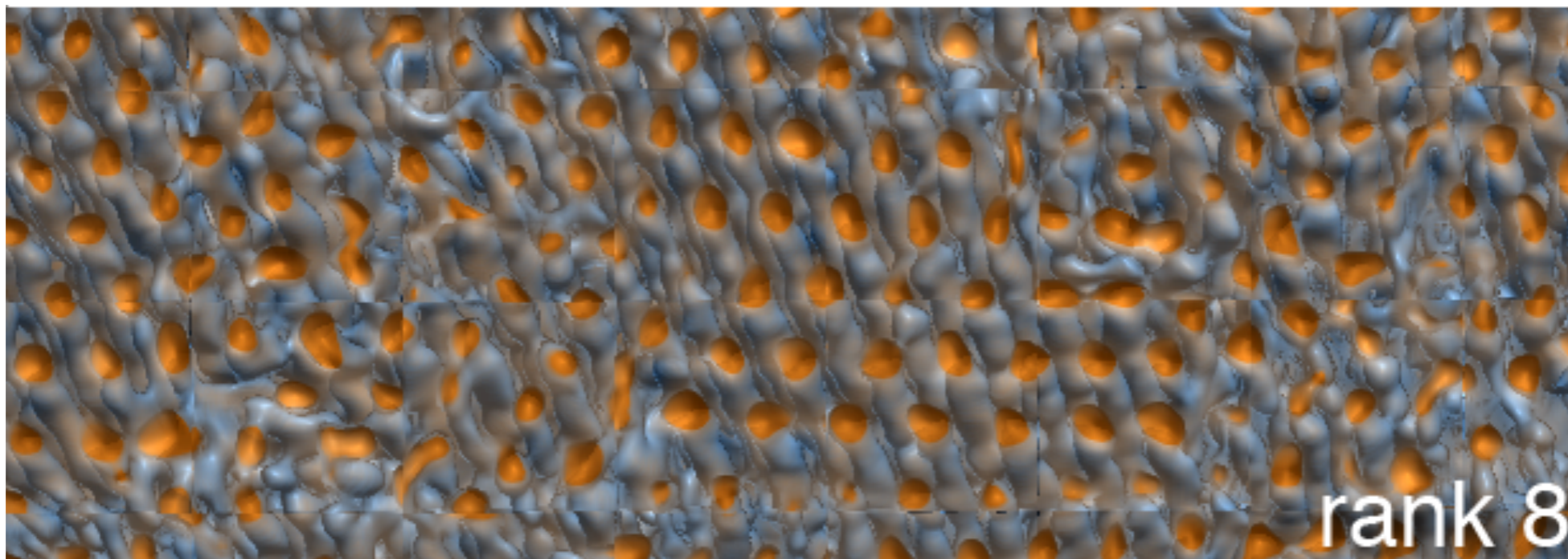
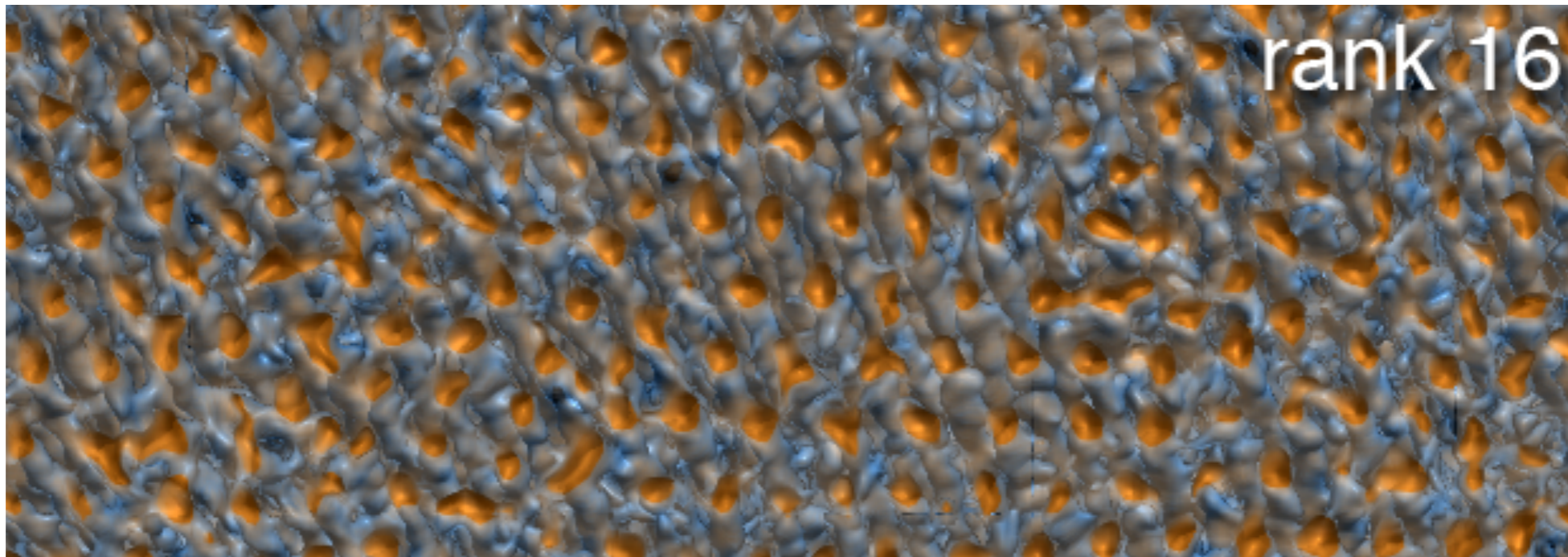
Multiresolution Direct Volume Rendering



Suter et al.. Interactive multiscale tensor reconstruction for multiresolution volume visualization. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2135–2143, December 2011.

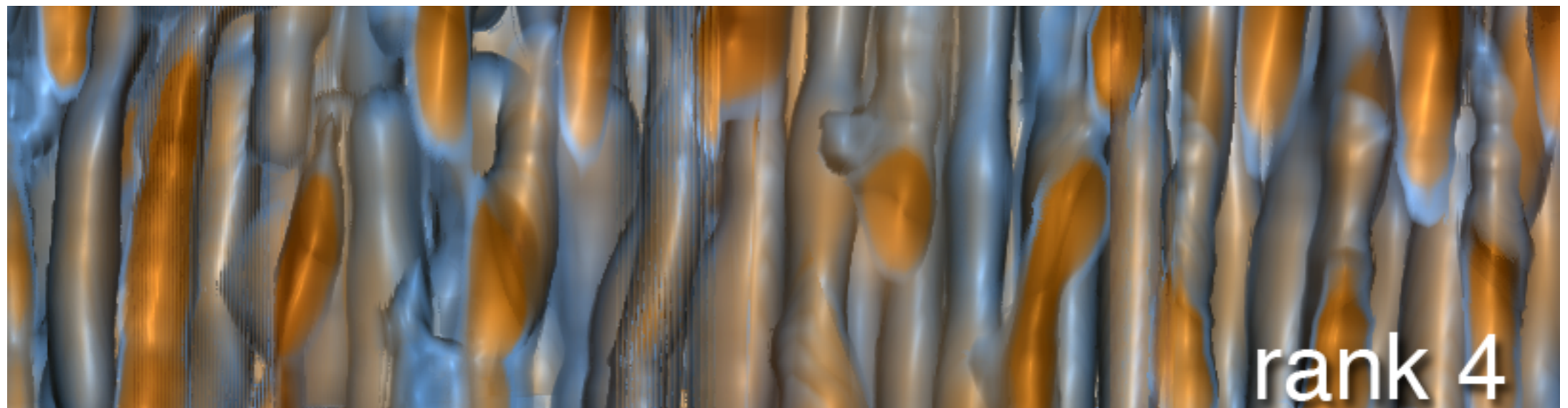
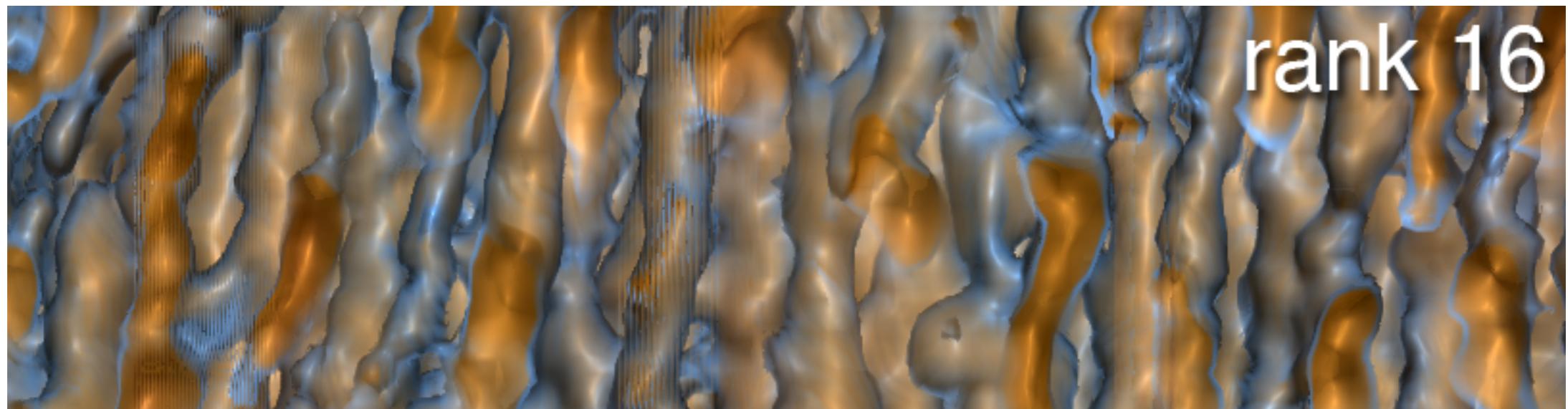
Rank-reducibility and Feature Extraction

[Suter et al., 2011]

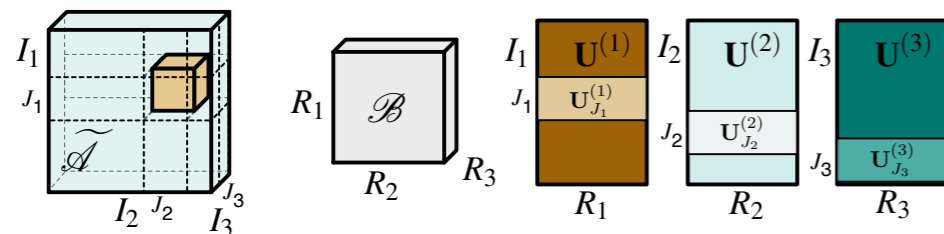
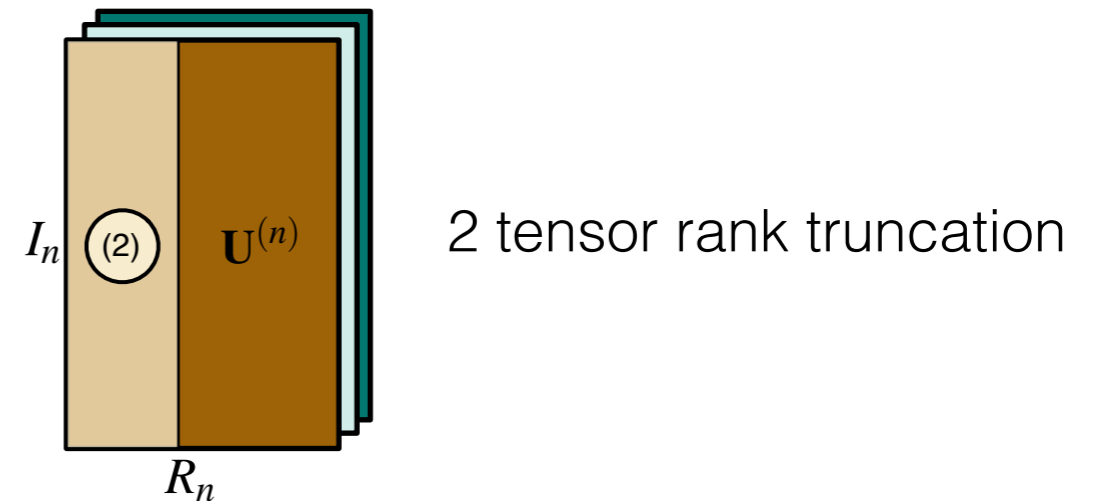
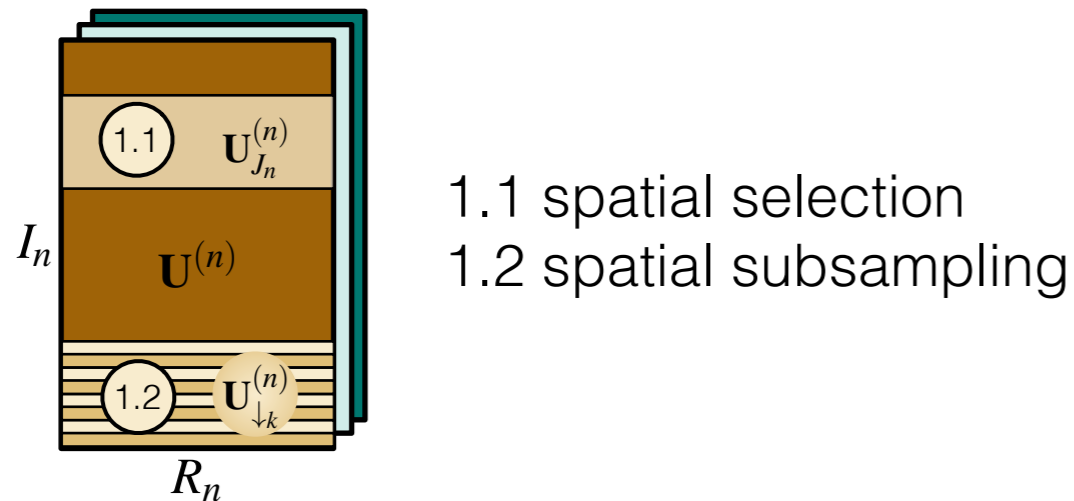


Rank-reducibility and Feature Extraction

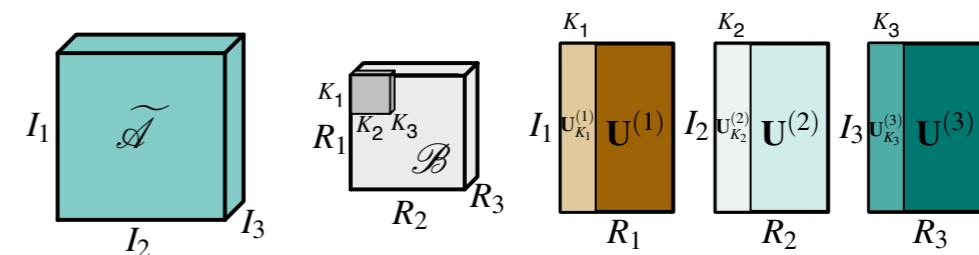
[Suter et al., 2011]



Recap: Tensor Bases and Properties

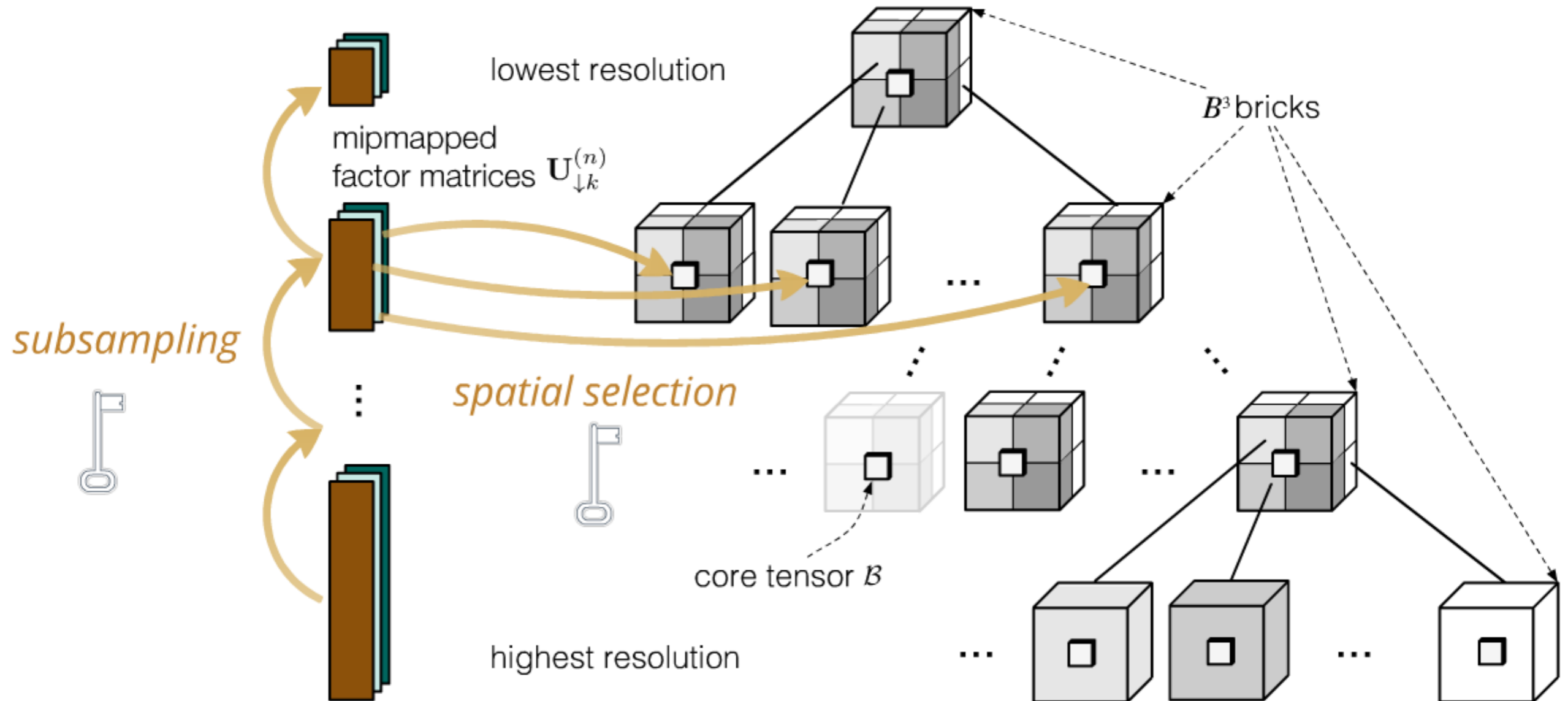


use for state-of-the-art
multiresolution volume rendering



use for **multiscale** volume feature
visualization

Multiresolution and Multiscale DVR



Suter, Makhinya and Pajarola. TAMRESH: Tensor approximation multiresolution hierarchy for interactive volume visualization. *Computer Graphics Forum*, 2013.

Multiscale Feature Visualization Demo (1)

Flower model

Model size: 1024^3

Screen resolution: 1024^2

Storage Costs of TA Hierarchy Models

- Theoretical costs
 - ▶ without empty space skipping
 - ▶ without pruning/thresholding of coefficients
- Assumptions
 - ▶ brick size $B = 64$
 - ▶ initial rank $R_{init} = 32$
- **Wu et al., 2008:**
- **Suter et al., 2011:** $\approx 0.17 \cdot I^3$
- **Suter et al., 2013:** $\approx 192 \cdot I + 0.14 \cdot I^3$

Quantization of TA Hierarchy Models

- Compact representation coefficients usually floating point numbers
- Quantize coefficients

	factor matrices	core tensors
Wu et al., 2008	8-bit	8...20-bit
Suter et al., 2011	16-bit	8-bit
Suter et al., 2013	32-bit*	8-bit

*no quantization

Conclusion

- Compact data representations in scientific visualization
 - ▶ Tucker models
- Multiscale feature extraction
 - ▶ tensor rank truncation
- Hierarchical (multiresolution) Tucker models
 - ▶ residual-based approach (pruning important) [Wu et al., 2008]
 - ▶ simple brick-based multiresolution model [Suter et al., 2011]
 - ▶ global bases; multiresolution and multiscalability [Suter et al., 2013]
- Compression via TA