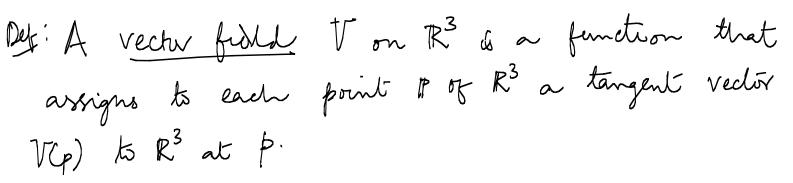
Vector field

_Note Title



4/16/2006

Roughly splaking, a retry field is put a
big collection of arrows, one at each point of
$$\mathbb{R}^3$$
.
 $(V + W)(\varphi) = V(\varphi) + W(\varphi)$
 $(fV)(\varphi) = f(\varphi) V(\varphi)$

Def Let
$$U_1 = (1,0,0)$$
, $U_2 = (0,1,0)$ and $U_3 = (0,0,1)$ be
the vector fields on \mathbb{R}^3 . for each pt. # of \mathbb{R}^3 . These
three vector fields collectually called the
NATURAL FRAME FIELD on \mathbb{R}^3
there are Three
impuly determined head-valued functions
 $V_{\overline{i}}, V_{\overline{2}}, V_{\overline{3}}$ on \mathbb{R}^3 such that
 $V = V_1 + V_2 U_2 + V_3 U_3$.
 $V_{\overline{i}}, V_{\overline{2i}}$ and V_3 are alled Euclinger for \overline{V} .

Directional Derivatives

Def. Let f be a dufferentiable real-valued
function on R³, and let V_p be a tangent
vector to R³. Then the number
V_p[f] =
$$\frac{d}{dt} (f(P+tv)) \Big|_{t=0}$$

is called the derivative of f Writ V_p
If V_p is a unit vector then V_p[f] is called
the directional derivative of f writ V_p.

Lemma
$$V_p = (V_1, V_2, V_3)$$
 is a tangent vector to \mathbb{R}^3
then $V_p[f] = \sum V_1 \frac{\partial f}{\partial X_1}(p)$

Derivative of
$$f$$
 with natural frame is
 $F = \frac{\partial f}{\partial x_{1}}, \frac{\partial f}{\partial x_{2}}, \frac{\partial f}{\partial x_{3}}$. This is a vector and
 $t = \frac{\partial f}{\partial x_{2}}, \frac{\partial f}{\partial x_{3}}, \frac{\partial f}{\partial x_{3}}$.
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 $t = \frac{\partial f}{\partial x_{2}}, \frac{\partial f}{\partial x_{3}}, \frac$

Thoras $l - (aV_{\mu} + bW_{\mu})(f) = aV_{\mu}(f) + bV_{\mu}(f)$ 2. $V_p [af tbg] = aV_p [f] + bV_p [g]$ 3. $V_p(fg) = V_p(f) \cdot g(p) + f(r) \cdot V_p(g)$ Directional Denvetive along a Vector field is malogous. COVARIANT DERIVATIVE Det. Let W be a vector field on R³ and let V be a tangent vector field to R³ at the point p. Then the covariant derivative of W Lost V is the tanget vector $\nabla_{v}W = W(p+tv)'(0)$ at the point P.

How to compute Covariant Derivatives ?

Example. $W = \chi^2 U_1 + \chi^2 U_3$ V = (-1, 0, 2) P = (2, 1, 0)P + tv = (2 - t, 1, 2t) $W(p_{t}tv) = (2-t)^{2}v_{1} + 2tV_{3}$ $\nabla_{v} W = W(p_{t}tv)'(o) = -4V_{1}(q) + 2V_{3}(q)$ ALTERNATE WAY OF WMPNTING COVARIANT DERIVATIVES $W = W_1 U_1 + W_2 U_2 + W_3 U_3$ M, WZ, and Wz are Scalar coordinate for. Compute the rate of change of these individual functions in the direction of V.

 $V[W_1]$, $V[W_2]$ and $V[W_3]$ The covariant derivative of the Wrt V is. Vector fuild W $+ v \left[\frac{w_2}{2} \right] \frac{v_{z}}{2} + v \left[\frac{w_3}{3} \right] \frac{v_3}{3}$ $\nabla_{V}W = V[W,]U_{I}$ $P_{\text{revious Example V = (-1, 0, 2)} P = (2, 1, 0)$ $\omega_1 = \chi^2 \qquad \omega_2 = 0 \qquad \omega_3 = \gamma_3^2$ $W_{l}' = \left(\frac{\partial W_{l}}{\partial \chi}, \frac{\partial W_{l}}{\partial \chi}, \frac{\partial W_{l}}{\partial \chi}\right) = \left(2\chi_{l}, 0, 0\right)$ $V[\omega_i] = V \cdot w' = -2\mathcal{X}$ Similarly $V[W_2] = 0 \quad V[W_3] = 2y$ $\nabla_{V}W = -2\chi U_{1} + 2\gamma U_{3}$ $(\nabla_{v}w)(p) = -4 \cup_{1}(p) + 2 \cup_{3}(p)$

Another Example $W = \chi U_1 + \chi^2 U_2 - 2 V_3$ (1, -1, 2) p = (1, 3, -1) $W_1' = (1, 0, 0) \quad W_2' = (2x, 0, 0)$ $w_3' = (0, 0, -23)$ $V[w_1] = l \quad V[w_2] = 2\pi \quad V[w_3] = -4\gamma$ $\nabla_{V} W = (1, 2\pi, -43)$ $(\nabla_{V} \psi)(P) = \psi, (P) + 2\psi_2(P) + 4\psi_3(P)$