Keywords: Mathematics Education, Post-primary Education, Technology, Contextualised Learning.

Abstract: There is ongoing debate about the quality of mathematics education at post-primary level. Research suggests that, while the capacity to use mathematics constructively is fundamental to the economies of the future, many graduates of the secondary-school system have a fragmented and de-contextualised view of the subject, leading to issues with engagement and motivation. In an attempt to address some of the difficulties associated with mathematics teaching and learning, the authors have developed a set of design principles for the creation of contextualised, collaborative and technology-mediated mathematics learning activities. This paper describes the implementation of two such activities. The study involved 24 students aged between 15 and 16 who engaged in the activities for 2.5 hours each day over a week long period. Initial results indicate that the interventions were pragmatic to implement in a classroom setting and were successful in addressing some of the issues in mathematics education evident from the literature.

1 INTRODUCTION

Research suggests that, while the capacity to use mathematics constructively will be fundamental to the economies of the future, the view that many graduates of the secondary-school system have of the subject is fragmented and lacking in context, leading to issues with engagement and motivation (Gross et al., 2009; Grossman, 2001). This study looks at how the affordances of readily available digital technology can be exploited to create mathematical activities that address common issues in mathematics education.

There is strong evidence in the literature that an approach to mathematics education encouraging contextualised, collaborative solving of mathematical problems is beneficial (Hoyles and Noss, 2009; Olive et al., 2010). Following an extensive review and analysis of the recent literature on technology-enhanced mathematics learning interventions, the authors have devised a set of guidelines to assist teachers in the design and delivery of such interventions, a number of which have been piloted in an experimental learning environment in the authors’ institution. Following from these pilot interventions, a larger scale set of activities has been implemented in a conventional school setting, the preliminary results of which will be discussed in this paper.

The overarching research in which this study is situated follows a design-based methodology (Anderson and Shattuck, 2012; Mor and Winters, 2007), in which a series of technology-mediated mathematical tasks are developed in tandem with the theory and principles that underpin them. The design principles for the activities are evolving from the ongoing literature review and classification process, in conjunction with analysis of empirical findings from teaching experiments in natural and exploratory settings.

This paper is consists of two main parts. In order to contextualise the current research within the broader field, a literature review and background to the current work is presented. The paper then describes a week-long intervention in a conventional co-educational school setting, involving 24 mixed-ability students. Preliminary findings from the intervention will be discussed, along with its impact on the design principles and future work.

2 BACKGROUND

In order to ground this research within the wider context, this section includes a literature review of the general issues in mathematics education, as well
as specific topics relating to the use of digital technology in the field. A synopsis of the development and analysis of the classification system, and the development of the design principles and related activities is also provided.

2.1 Issues in Mathematics Education

There is an unfortunately prevalent view of mathematics as a collection of unrelated facts and rules, and a related belief that learning mathematics involves memorisation and execution of procedures leading to unique, correct answers (Ernest, 1997); an assumption that mathematics is “hard, right or wrong, routinised and boring” (Noss and Hoyles, 1996, p. 223). This formal, abstract and assessment driven approach to mathematics education remains dominant in many countries (Ozdamli et al., 2013) contributing to behaviourist and didactic tendencies in teaching and learning, with an emphasis on content and procedure over literacy and understanding. In this context, mathematical creativity is not prized and students are rarely encouraged to seek out their own alternative solutions (Dede, 2010). The authority of the teacher is perceived as absolute, their job to transmit information to the students.

Efforts to address some of these issues have met with limited success. Attempts to introduce problem-solving and realistic context to mathematics teaching and learning are particularly pertinent to this research. However, as Boaler (1993) suggests, such problems are frequently uninteresting from the point of view of the students as they are generally formulated in such a way as to be routine problems with just a veneer of the ‘real-world’. In an attempt to reduce complexity, the activities are overly well-defined, furnishing all of the information required to solve the problem, without excess. The learner is reduced to following the standard procedure of inserting data into appropriate formulae in an attempt to get the ‘correct’ answer (Dede, 2010). The authority of the teacher is perceived as absolute, their job to transmit information to the students.

2.2 ICT and Mathematics Education

The use of digital technologies in mathematics education has the capacity to open up diverse pathways for students to construct and engage with mathematical knowledge, embedding the subject in authentic contexts and returning the agency to create meaning to the students (Drijvers, Mariotti, Olive, & Sacristán, 2010; Olive et al., 2010).

Noss and Hoyles (1996) propose that technology has the potential to bring meaningful mathematics into the classroom. It can facilitate an emphasis on practical applications of mathematics, through modelling, visualisation, manipulation and more complex scenarios (Olive et al., 2010).

Many authors contend however, that although use of technology in the classroom is increasing, its potential to enhance the learning experience lags behind its implementation in the classroom (Geiger et al., 2010; Hoyles and Lagrange, 2010). While students may engage in the creative use of digital technologies on a daily basis, they do so less frequently in an educational context (Oldknow, 2009; Pimm and Johnston-Wilder, 2004).

Jonassen, Carr, and Yueh (1998) contrast technologies that attempt to instruct the learner, with what they describe as mindtools - technological tools that students learn with, rather than from – which support knowledge construction by engaging them in critical thinking. Thus technology becomes a mediator of the learning experience, facilitating reflective, discursive and problem-solving skills.

In this research, we are attempting to facilitate the use of digital technology as ‘mindtools’ to encourage the development of the desired skill set by scaffolding implementation through the emerging design principles.

2.3 Analysis of Empirical Interventions

At the outset of the research process, it became clear that a system of classification would be beneficial in order to put a framework on the current trends in the literature relating to technology usage in mathematics education.

An ongoing, systematic review of recent literature in which technology interventions in mathematics education are described is used as the foundation of such a system of classification, a detailed analysis of which can be found in (Bray and Tangney, 2013b). Trends emerging from the analysis of the classification are used in conjunction with a broader literature review, to inform a set of design principles for the development of interventions in the field.

Through the classification it is evident that a wide range of technologies are being researched in different environments, with different agendas and from varying theoretical standpoints. What most interventions have in common is a trend towards social constructivism and a desire to create engaging environments in which the technology is used to increase the students’ interest, motivation and performance. The pervasive perception of mathematics education emerging from the papers
focuses on understanding of relations, processes and purposes, as opposed to the requirement to learn a fixed body of knowledge. There is a move towards connection, coherency and context as important aspects of mathematics education that can be facilitated by technology.

2.4 Emerging Design Principles

Analysis of the classified papers, along with a general literature review, provides the theoretical foundations for a set of design principles for the development of innovative, technology-mediated, mathematical activities. Using a first iteration of the design principles, a number of activities have been devised and trialled in an exploratory environment. The results of these pilot studies have fed back into theoretical foundations of the research, leading to refinement of the classification and design principles. Our intention in developing these guidelines and activities is to increase student engagement and motivation with mathematics and to increase teacher awareness of how to support learning within these scenarios.

The design principles resonate with a view of mathematics as a problem-solving activity and of mathematics education as involving students in constructing their knowledge via the social formulation and solution of problems. A need for the development of tasks that are transformed through the use of technology, providing contexts that are relevant and of interest to the students, and which have compelling goals is evident (Confrey et al., 2010; Laborde, 2002; Oldknow, 2009). Technologies that outsource the burden of computation have proven to be an interesting area of research, not only improving speed and accuracy of students engaged in procedural tasks, but also allowing increasing emphasis to be placed on meaning as opposed to routine operation (Geiger et al., 2010; Oates, 2011). The use of a variety of accessible, free technologies is an important issue, not only due to matters of equity, but also to engender flexibility amongst students and teachers (Oldknow, 2009; Sinclair et al., 2010).

2.5 Initial Learning Activities

A number of activities have been designed in accordance with the design principles, and have been piloted with groups of students and teachers in an exploratory learning centre, Bridge21, at the authors’ institution. The centre is designed to support a model of collaborative, technology-mediated and project-based learning (Lawlor et al., 2010). The teacher is seen as orchestrator rather than director of the learning, building on a model of peer learning and collaboration originating in the patrol system of the World Scout Movement (Bénard, 2002). Post-primary students are released from school to attend workshops in the centre, of between four and five hours duration.

The activities that have been tested to date include The Human Catapult (projectile motion, functions, angles and velocity) and The Scale Activity (estimation, orders of magnitude and scientific notation) described in (Bray and Tangney, 2013a), as well as Probability and Plinko (independent events, normal distribution, Pascal’s triangle, probability, binomial distribution), the Pond Filling Activity (problem-solving, estimation and volume) (Tangney and Bray, 2013), and the Barbie Bungee (collecting, representing and analysis of data, linear functions, line of best fit, correlation, extrapolation). The interventions have provided data relating to the practicality of the tasks and a starting point from which to begin the iterative process of development.

The results of the pilot interventions have provided the justification for further investigation in authentic classroom environments. This study reports on initial trials in an actual classroom setting.

3 THE INTERVENTION

The school in which the study took place is a co-educational private school in an urban area and is one of a network of schools cooperating with our institution in an attempt to adapt the Bridge21 model for use in mainstream schools. These schools are favourably disposed towards a collaborative, technology-mediated approach. In addition, participating students have had prior exposure to workshops in which they have been introduced to the Bridge21 model of learning, thus increasing their understanding of the processes involved in teamwork and project-based learning. When it comes to tackling the mathematical activities, they should therefore be well versed in the methodology and in a position to concentrate on the task.

The school in this study has re-modelled its approach to teaching and learning in line with the Bridge21 methodology. In light of this, the year 10 (age 15/16) timetable has been restructured in order to accommodate a 2.5 hour block of curriculum-related project work in the middle of the day. For the Contextual Mathematics intervention the 1st author...
had access to students for this project block each day for one week. During this period, the author acted as primary teacher, or facilitator, with one classroom assistant. The class consisted of 24 students (12 male and 12 female), of mixed ability, who were assigned to 6 groups of 4 students each. The groups were assigned in order to balance abilities and gender. The environment was made up of two adjoining rooms with double doors between them. Each team had an allocated area, or workstation, with access to at least one computer, where they could work together. Laptops, cameras and other props were provided by the researchers. Students had permission to leave the school premises when the activity required.

3.1 Methodology

As described in the introduction, the overarching research project employs a design-based methodology (Anderson and Shattuck, 2012; Mor and Winters, 2007) whereby the mathematical activities and the theory that underpins them are developed in a complimentary and iterative manner. Data from individual case studies, collected by way of observation, semi-structured interview and questionnaires, helps to inform and refine both the design principles and the activities themselves.

The Mathematics and Technology Attitudes Scale (MTAS) (Pierce et al., 2007) was used as a pre- and post-questionnaire, giving a quantitative measure of confidence levels in mathematics and technology, behavioural engagement, affective engagement, and attitudes to using technology in mathematics. Qualitative data was gathered from student journals, written comments and a semi-structured interview with 5 of the 6 team leaders. At this stage only preliminary results are available from the qualitative data as the process of coding and theming is in its early stages.

3.2 Outline of the Activities

In this section, an outline of the weeks’ activities is provided. Every day followed the same general structure, based on the learning model developed in the Bridge21. Each session began with an initial plenary discussion in which previous work was reviewed and the mathematical problems and activities for the day were presented. This was followed by a team planning, after which team-leaders met to discuss possible solution strategies with the facilitator and assistant. Once the plans were approved, the groups were free to implement them. As the teams worked, the facilitators interacted with the students, scaffolding their exploration of the mathematics and technology. At the end of the session, each group presented their work, discussing what individual team members had been responsible for, what had been accomplished, and what mathematics they had understood. After a final whole group discussion, take-home problems were assigned. These were short questions designed to be thought provoking and interesting, and requiring the students to be creative with their solving strategies.

The first day consisted of warm-ups, team-building activities and Fermi-type problems. These are exercises in estimation and approximation, encouraging problem-solving and mathematical creativity. The ‘correct’ answer is not the primary goal, and many approaches to the solution are acceptable. Examples used include the following.
- Estimate the number of blades of grass in the local park.
- Estimate the average walking speed of people outside the local park.
- Estimate how many seconds old you are.

The teams had permission to use the internet, giving them access to Google maps, grid overlay tools etc. Each team was also furnished with a measuring tape and a camera.

Day 2 marked the beginning of the program of activities that were the primary focus of this study. Although the concept of a Barbie Bungee is not a new in mathematics education, embedding it in a loosely scaffolded, technology-mediated and team-based environment has lent it a novel and innovative perspective.

Each group was provided with a Barbie doll, a box of rubber bands, a camera, a laptop with the free video analysis software Kinovea and a spreadsheet program. They were asked to estimate the number of rubber bands needed to give Barbie an exhilarating, but safe jump, from a first floor window. Trial and error was not permitted, and they were not initially allowed to leave the room to measure the distance of the fall. Particular incentive was given by making the testing of their hypotheses into a competition. The groups used diverse methods of tying the bands and adding weights to the dolls. All but one of the teams made use of the available digital technology to video the bouncing Barbie in order to accurately capture the distance she dropped. Each group recorded their data in a spreadsheet and used the capabilities of the technology to create a scatter plot and generate a line of best fit. Most of the teams had reached this point in time for the wrap-up session at
the end of the day.

Day 3 began with a very interesting discussion about functions, correlation, causality and extrapolation. The groups then estimated the distance the Barbie would need to drop from the first floor window and returned to the functions that described their line of best fit. Once the dolls were attached to their bungees, the knockout competition began and two by two the teams competed to see whose doll got closest to the ground without hitting it.

Figure 1: Barbie Bungee Competition.

Once the winning team was decided and the prizes were distributed, the discussion regarding the next activity began.

The Human Catapult activity is an investigation into projectile motion. Teams use an oversized slingshot, foam balls, cameras and the free video analysis software Tracker (www.cabrillo.edu/~dbrown/tracker), and GeoGebra (www.geogebra.org), to investigate concepts such as functions, angles, rates of change and velocity.

After a plenary session in which the optimal approach to video recording for the purposes of generating quadratic functions was discussed, the groups spent the second half of the 3rd day in the local park recording their team members using the catapult to fire a foam ball.

The plenary session that began the 4th day highlighted the mathematical connections that underpin the Barbie and Catapult activities. Although the methods of data collection differed – manual measurement and plotting of points on a graph, or automatically generated functions through frame-by-frame video analysis – the approach of using the line/parabola of best fit for modelling and generalisation was common to both activities. In addition, the concept of correlation and causality that had been introduced with the Barbie activity was explored in significant depth through the graphs of the functions generated by the catapult. The initial graph discussed was the pictorial representation of the flight of the ball through the air, in which the x-axis represents horizontal distance and the y-axis represents height.

Figure 2: Tracker Generation of Initial Graph.

On discussing whether there was a causal relationship between these two variables, one of the students remarked: “well, if the distance is counted as time there is”, which allowed for the deconstruction of the original graph into the two more meaningful graphs of height with respect to time and horizontal distance with respect to time.

The groups used the video analysis and best-fit functionality in Tracker to generate relevant functions, which were then analysed in GeoGebra. After initial technical difficulties, most of the teams managed to generate the functions and begin their modelling. While calculating the angle of projection and maximum height were straightforward tasks, estimation of the initial velocity of the ball is quite an involved concept and this part of the activity was left for the final day.

Once the concept of initial velocity and possible approaches to its calculation were explained, the teams were given time to try to work it out before using a simulation on phet.colorado.edu to gauge the accuracy of their mathematical model. Once again, a competition was used to incentivise the efforts and a final showdown, in which the actual distances were compared against the simulated distances, was used to judge the endeavour. Once preparation of the final presentations was complete, the groups took turns to talk about what they had achieved.

3.3 Preliminary Analysis of Results

While results presented in this section are in the early stages, preliminary analysis indicates some interesting outcomes.

21 of the 24 students completed both a pre- and post-questionnaire, which was designed to highlight changes in their attitudes to mathematics and technology, and their levels of behavioural and
affective engagement over the course of the intervention. The questionnaire used was the Mathematics and Technology Attitudes Scale (Pierce et al., 2007) – a 20 item questionnaire with a likert-type scoring system that measures mathematical confidence, technological confidence, behavioural engagement, affective engagement and attitude to using technology in mathematics. There was a small increase in behavioural engagement and in attitudes to using technology in mathematics, and a slight decrease in mathematical and technological confidence. However, a 6% increase in affective engagement was recorded.

As short-term significant changes are hard to achieve, and these changes have not yet been tested for statistical significance. However, from the qualitative data it seems that the drop in confidence levels relates to the change from the typical, formulaic approach to mathematics education to the use of messy data with no absolute “correct” answer to the activities.

At this stage of the analysis, we have decided to use a word-cloud of the most frequently recorded 50 words of 4 or more letters to provide a feel for the qualitative data that has emerged from the intervention. This graphical representation of word frequency is not meant as a substitute for traditional content analysis – which is ongoing at the time of writing – but as a visually rich way to enable readers to get a feel for the data at hand (Joubert, 2012; McNaught and Lam, 2010). In a word-cloud the size of the word relates to the number of times it occurs. The data was gathered from student post-questionnaire comments, individual journals, and from the transcription of a 25 minute, semi-structured focus group interview. Before running the word frequency analysis on the data, usage of the word “like” as a vocalised pause was removed from the transcript of the interview so that it would not pollute the data. This usage of the word is common among teenagers as a meaningless interjection, to keep conversation flowing.

The relatively large size of positive attitudinal words such as “like” (used to represent enjoyment), “enjoy” and “interesting” support the increase in affective engagement recorded in the quantitative data. Additional support is found in quotes such as:

- “I liked this week; it did not feel like maths in a way, it felt like fun. It felt different from school maths but I still learned things.”
- “I am/ shaken in my absolute use of the term ‘hate’ [relating to mathematics] and more on the side of ‘mildly dislike’.”

Even students with negative initial attitudes seemed to have a positive experience:

- “It was definitely better than normal school maths. It was far more engaging.”
- “I felt that leaving us to it and letting us go out was great.”
- “I liked this week; it did not feel like maths in a way, it felt like fun. It felt different from school maths but I still learned things.”

The focus group interview involved team leaders from five of the six groups (one team was unavailable) and the 1st author, and shed light on many of the positive and negative aspects of the intervention. One student felt that the aim of the approach was to create a more engaging and involving way to learn mathematics, encouraging students to “think outside the box”. When queried as to whether he meant problem solving, he replied: “it’s not just simple problem solving, like when you get this big long-winded question, and... you know it’s simultaneous equations, or you know it’s going to be graphs. This is like, it doesn’t tell you what it is, you just have to figure it out yourself”. Another student found using the facility of the technology to outsource the calculation was very beneficial: “using the computers was really handy, because it meant that I could understand it and have fun with it, without having to stress about getting it wrong”. All of the students agreed that the emphasis was more on understanding of concepts as opposed to procedures and content. When questioned about the development of new understanding, a number of them pointed out that prior to the intervention, they had not realised the extent of relationships between different areas of mathematics, and how, in many cases, what are often presented as diverse topics are simply different modes of representation. Others had developed a deeper understanding (or in some cases “an” understanding) of functions.

There were contrasting reactions to the usage of technology in the groups. Some of the students felt
that it gave them freedom to understand and manipulate the mathematics, while others preferred more concrete, hands-on activities: “I didn't like... when we were using GeoGebra. That's why I liked the Barbie thing, because you can hold it. I liked seeing it in my hands and being able to pull it and see what happens and that, but on a computer it seems very abstract”.

4 DISCUSSION

While digital technology has the potential to open new routes for students to construct and comprehend mathematical knowledge and new approaches to problem-solving, this requires a change in the pedagogical approach in the classroom in terms of student engagement with learning (Drijvers et al., 2010). Olive et al. (2010) highlight that “it is not the technology itself that facilitates new knowledge and practice, but technology’s affordances for development of tasks and processes that forge new pathways” (p154).

The need to conduct research into the design and development of tasks and activities that provide engaging environments, in which the mathematics are seen as relevant by the students, with goals that they find compelling (Confrey et al., 2010; Laborde, 2002; Oldknow, 2009) is the motivating factor for this work. In this study, technology has facilitated research, data gathering and analysis, outsourcing of computation and mathematical modelling, all of which have permitted a level of engagement with mathematical concepts that would not otherwise have been possible. This is reflected in the increase in affective engagement recorded in the MTAS scores, but perhaps more significant is the sense of student ownership and the understanding of connections, mathematical context and relevance that is evident from the students’ qualitative responses.

Kieran and Drijvers (2006) contend that mathematical tasks that make use of technology should not be studied without also paying careful attention to the classroom environment and the role of the teacher. Flexibility with regard to routine and environment are necessary in order to fully exploit the potential of technology in the teaching and learning of mathematics; the block structuring of the timetable in the School in which the study took place facilitated real student engagement with the activities. If the activities were to be conducted within the confines of a more conventional timetable, with periods of between 35 and 90 minutes, the experience would have been more fractured and, while it may still be possible, it is unlikely that the same level of engagement would have been achieved.

Means (2010) points out that higher learning gains are associated with classrooms in which an established routine is in place for moving between technology-mediated and traditional activities. Orchestration of the classroom and technological difficulties relating to network access and up-to-date software emerged as an issue that needs serious consideration and contingency planning before further interventions of this kind are undertaken.

The week-long intervention in an authentic school setting has provided a positive view of the approach to integrating technology in mathematics education proposed in this research. The initial results indicate that there is real potential for increased engagement and conceptual understanding emerging from participation with activities designed in accordance with the design principles.

REFERENCES


Bray, A., & Tangney, B. (2013a). The Human Catapult and Other Stories – Adventures with Technology in Mathematics Education. 11th International Conference on Technology in Mathematics Teaching (ICMT11), 77 - 83.


Barbie Bungee Jumping, Technology and Contextualised Learning of Mathematics

Technology - Rethinking the Terrain: The 17th ICMI Study (Vol. 13, pp. 81 - 88): Springer.


