# Foundation of Computer Science 1

Revision 1
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**Problem 1.** Consider an information system $X = \{x_1, x_2, x_3, x_4, x_5\}$ with 5 messages $x_1, \ldots, x_5$. The probabilities of transmitting the messages $x_1, \ldots, x_5$ are, respectively, $p_1 = 0.5$, $p_2 = 0.25$, $p_3 = 0.125$, $p_4 = 0.0625$, and $p_5 = 0.0625$.
What is the minimal length of a word code in a binary encoding of $X$?

**Problem 2.** Let $p$, $q$ and $r$ be propositional formulas.

**2.1** By computing truth tables, show that $p \wedge q \wedge (\neg p \vee \neg q)$ and $\neg p \wedge \neg q \wedge (p \vee q)$ are logically equivalent.

**2.2** Prove that $(p \implies r) \wedge (q \implies r)$ and $(p \vee q) \implies r$ are logically equivalent. Your proof should rely on using equivalent transformations rules.

**2.3** Consider the propositional formula

$$(p \vee q) \wedge (\neg p \vee r) \implies (q \vee r).$$

By using those simplification rules which you consider appropriate, show that it is a tautology.

**2.4** Consider the propositional formula

$$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r)$$

Is this formula satisfiable? Is it a tautology? Is it a contradiction?

**Problem 3.** Let $x$ denote a variable, and $P$, $Q$ and $R$ be predicate formulas such that the variable $x$ does not occur as a free variable in $Q$. Assume that the domain of variables is nonempty.

**3.1** Prove using equivalent transformation rules the following formula:

$$((\exists x :: P(x)) \implies Q) \implies ((\forall x :: P(x)) \implies Q)$$

**3.2** Establish the logical equivalence:

$$\forall x :: (P(x) \implies Q) \Leftrightarrow (\exists x :: P(x)) \implies Q$$

**3.3** Suppose that the domain of $x$ consists of $-4$, $-2$, 4, and 8. Express the statements below without using quantifiers, instead using only negations, conjunctions and disjunctions.

(a) $\forall x :: P(x)$;

(b) $\exists x :: (\neg P(x)) \wedge \forall x :: ((x \neq 2) \implies P(x))$

**3.4** What is the truth value of the formula $(\text{Anz } x :: P(x)) = 1 \implies \exists x :: P(x)$?

**3.5** Determine whether $\forall x :: (P(x) \implies R(x))$ and $\forall x :: P(x) \implies \forall x :: R(x)$ are logically equivalent. Justify your answer!