Commodity Trading
Option Pricing

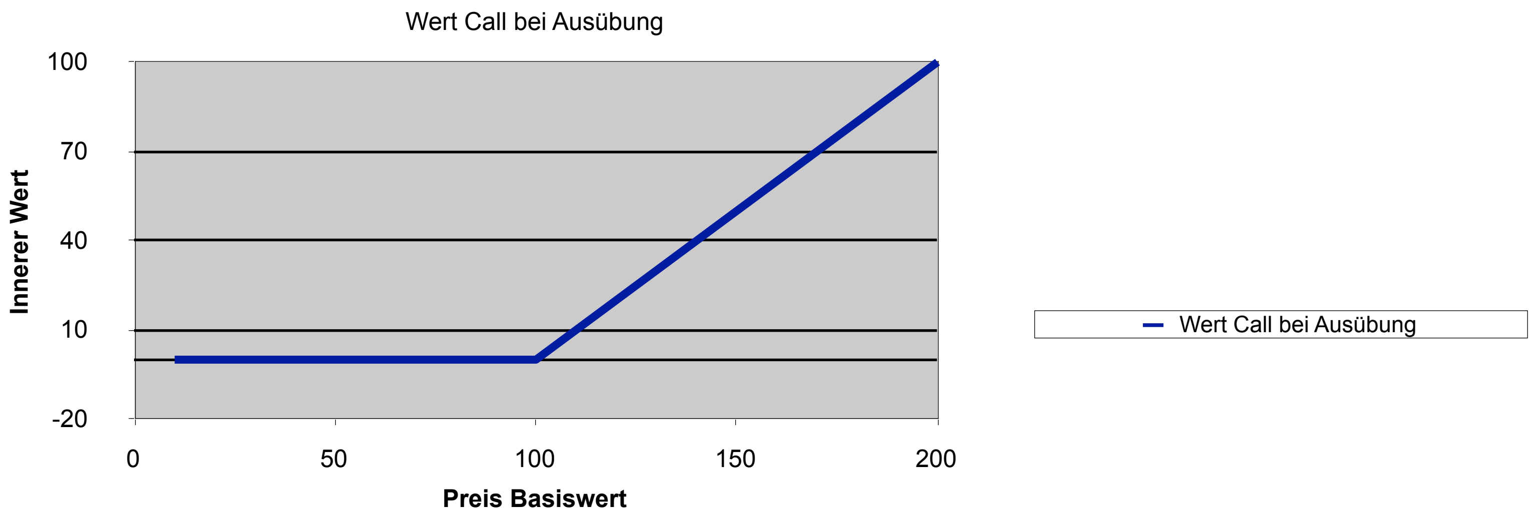
28.3.2011

Optionpricing

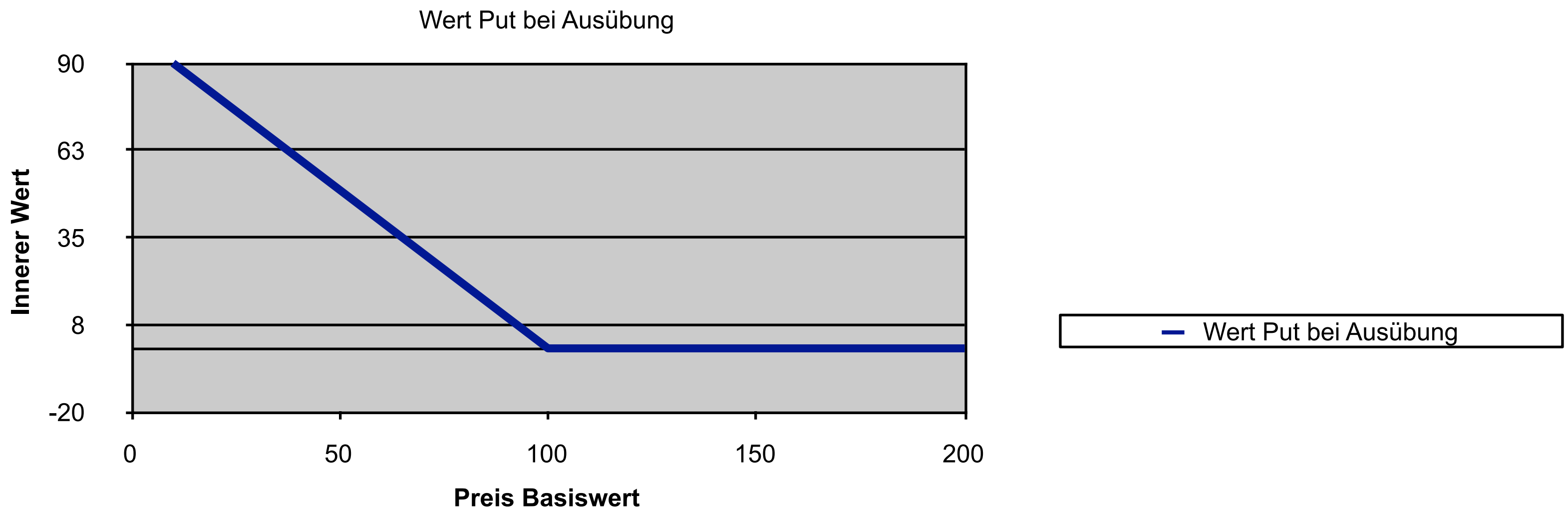
- Relative Pricing ➔ Comparativeness
- Accessible through probabilistic approach ➔ Which is the right probability distribution?
- Accessible through “arbitrage free” approach ➔ Practicability?

Plain Vanilla Option

- A Call is the right to BUY the underlying instrument at the expiration date at the STRIKE price. The value at expiration is the greater of (price of the underlying the that date minus strike price) and zero
- A Put is the right to SELL the underlying instrument at the expiration date at the STRIKE price. The value at expiration is the greater of (strike price minus price of the underlying the that date) and zero



Wert Call bei Ausübung
100
70
40
10
-20
Innerer Wert
0
50
100
150
200
Preis Basiswert
Wert Call bei Ausübung



Wert Put bei Ausübung
90
63
35
8
-20
Innerer Wert
0
50
100
150
200
Preis Basiswert
Wert Put bei Ausübung

Probabilistic Approach

- What is the value of call strike 3 on a dice, one roll?

➔ Know Probability Distribution!

Probabilistic Approach

- Sum of all payout multiplied by there probability over all events:

  0*1/6 + 0*1/6 + 0*1/6 + 1*1/6 + 2*1/6 + 3*1/6

Arbitrage Free Approach

- Situation A: An Asset will be worth either 20% more or 10% less, tomorrow
- Situation B: An Asset will be worth either 10% more or 40% less, tomorrow
- Which Call at-the-money (Strike today's price) is more valuable?

- Idea from Didier Cossin, IMD

Pricing the Calls

- Assumption: one can buy or sell both underlyings and call freely and without fees or interest rate (the later make no substantial difference). Assume Price underlying 100.
- Suppose the price at the current market is very low, buy 1000 calls and sell Δ * 1000 underlyings, such that in both cases you receive the <u>same profit</u>.
- If you manage to do so, you can make a sure profit as long the price is too low.

Case A

- Price Underlying 100, Price Call C
- Buy 1000 Calls - pay 1000*C
- sell Δ * 1000 underlyings - receive Δ * 100’000
- Upper Case Profit=1000*(20–C)–Δ*20’000
- Lower Case Profit=1000*(–C) + Δ*10’000

Case B

- Price Underlying 100, Price Call C
- Buy 1000 Calls – pay 1000*C
- sell Δ * 1000 underlyings – receive Δ * 100'000
- Upper Case Profit=1000*(10–C)–Δ*10'000
- Lower Case Profit=1000*(–C) + Δ*40'000

General Case

- Price Underlying 100 can be either X up (100+X) or Y down (100–Y), Price Call C
- Buy 1000 Calls – pay 1000*C
- sell Δ * 1000 underlyings – receive Δ * 100'000
- Upper Case Profit=1000*(X–C)– Δ*X*1000
- Lower Case Profit=1000*(–C) + Δ*Y*1000
- Equal if $\Delta = X/(X+Y)$

Discussion (I)

- Option prices Volatility not expectation!

  (TAI????)

- Independent of probabilistic view? Check X>0 and Y<0. ➔

  Case B: The fact that the current price is much closer to the upper case than to the lower case implies that the

Discussion (II)

- Probabilistic View p:= probability that price increases (upper case) / 1-p:= probability that price decreases (lower case)
- Case A: $p*120 + (1-p)*90 = 100$
  ➔ $p = 1/3$; $1-p= 2/3$
- Case B: $p*110 + (1-p)*60 = 100$
  ➔ $p = 4/5$; $1-p= 1/5$

Probability = Model for

- Randomness ⇔Uncertainty
- Few thing are truly random, lots of influence are not detectable in reasonable time

Black Scholes Formula

$C = S * \mathrm{erf}(d_1) - K * e^{-rt} * \mathrm{erf}(d_1 - \sigma \sqrt{t})$

Where $d_1 = (\ln(S/(K * e^{-rt})) + 0.5 * t * \sigma^2) / \sigma \sqrt{t}$

C: Price Call

S: Forward Price Underlying

K: Strike

$K * e^{-rt}$: today‘s price of Strike

t: time to expiration

σ: implied volatility

Δ = Hedge Ratio

$\Delta = \text{erf}(d_1)$

whereas $d_1 = (\ln(S/(K^* e^{-rt)}) + 0.5 *t*\sigma^2) / \sigma \sqrt{t}$

Price Call / Delta vs. Preis Underlying